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A Spatial-Temporal Optimization Approach to Watershed Management:
AMD Treatment in the Cheat River Watershed, WV

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1. Introduction

Since passage of the Clean Water Act in 1972, significant improvements have been made in municipal wastewater treatment and industrial point sources pollution regulation with the goal of eliminating all discharges of pollutants so as to have fishable and swimmable waters (Boyd, 2000). Section 303(d) of the Act requires every state to list its polluted water bodies and establish priorities for clean up regardless of the sources. A watershed restoration action plan called a "Total Maximum Daily Load" (TMDL) for each impaired watershed also requires states to distribute pollutant reduction responsibilities among all point and nonpoint sources. Until the late 1990s, the TMDL plan went largely unused since most state efforts focused on point source pollution under the National Pollutant Discharge Elimination System (NPDES) (Elder, Killam and Koberstein, 1999). More than half of the 2,000 assessed watersheds in the United States still remain impaired due to pollution from point and, especially, nonpoint sources which include runoff from urban and suburban areas, agricultural lands, timber harvesting fields, coal mining sites, and others (National Wildlife Federation, 1999).

Facing the changing problems, the United States Environmental Protection Agency (USEPA) is developing an overall water quality-based approach (i.e., watershed management) instead of the previous technology-based point-by-point control to improve water quality. Watershed management incorporates input from all stakeholders including government agencies, industry, local organizations, and special interest groups within hydrological-defined geographic areas. This is a consensus-based approach designed to gain support from all interested parties and implicitly considers spatial interrelationships among natural ecosystems, anthropogenic forces, and the underlying physical system (Fletcher, Fuller, and Phipps, 2001). While this approach allows for pollution control by least-cost methods, information on the spatial and temporal dynamics provides additional information that allows informed decisions by such stakeholder groups or other management agencies.

However, most studies of water quality management concentrate on the inter-temporal allocation problem (for example, see Liebman and Lynn, 1966; Markris, 2001; and Opaluch, 1981), or, more recently, the spatial dynamics (Funk III, 1993; Greiner and Cacho, 2001; and Ali, 2002), but not both. Much of the literature focuses on spatial-temporal dynamics in fields such as invasion species, landfills, hedonic prices for environmental goods, dynamic equilibrium in coal markets, transportation, climatology, biological population, real estate, and infectious disease instead of water quality management (Burnett, Kaiser, and Roumasset, 2007; Gaudet, Moreaux, and Salant, 1998; Riddel, 2001;

Labys, Takayama, and Uri, 1989; Zawack and Thompson, 1983; Jagger, Niu, and Elsner, 2002; Renshaw, 1993; Pace, et al., 2000; and Deal, et al., 1999).

Several empirical studies involve both time and space dimensions in water allocation. Ejeta (2000) formulated a mixed integer nonlinear programming model that incorporates timing water supply capacity and allocation for the Rio Grande in New Mexico and Texas. Brozovic, Sunding, and Zilberman (2002) presented a model for the extraction of a path-dependent groundwater resource by spatially distributed users. Noel (1979) developed a linear quadratic model to determine the social optimum spatial and temporal allocation of water resources over a thirty-year planning horizon in Yolo County, California. In Noel's linear quadratic model, "the criterion function is an explicit measure of social welfare and is composed of" "producer and consumer surpluses and social surplus." Two sets of constraints are hydrologic constraints "estimated from a groundwater hydrology model and reservoir hydrology study," and "physical and institutional restrictions on water storage capacities, groundwater pumpages, and surface water diversions." The dynamic model is then used to "analyze the impact of four different energy cost scenarios on the socially optimal allocation of ground and surface water" (Noel, 1979).

In the theoretical area, Conrad and Clark (1987) presented a model where a residual discharged at one location may be transported to another location by wind or diffusion. Their model considers temporal and spatial dimensions at the same time and discusses the current-value Hamiltonian; Chiang (1992) added state-space constraints to the optimization problem and developed an alternative approach to modify the solution procedure. Studies that combine the spatial-temporal issues in an optimization model for water quality management are still rare.

This paper presents an approach to water quality analysis that incorporates both spatial and temporal dynamics in a watershed framework. The acid mine drainage (AMD) problem in the Cheat River watershed, West Virginia serves as a case study and provides an opportunity to test the modeling approach developed. Three subwatersheds – the Muddy Creek, Little Sandy Creek, and Albright Region subwatersheds – are selected to conduct the empirical analyses.

The rest of the paper is organized into four parts. The next section presents the general spatial-temporal dynamic optimization model that maximizes the present ecological value of the water resources of the watershed. The following section specifies a specific model to demonstrate the AMD treatment problem in the Cheat River watershed as a case study. The next section discusses the empirical results of three selected subwatersheds – Muddy Creek, Little Sandy Creek, and the Albright Region subwatersheds. The conclusions follow.

2. The General Spatial-Temporal Optimization Model Setup

For expository purposes, the following concepts are important. The initial segment of a stream from the source to the first confluence with another stream is called a headwater stream; the point where two or more streams join is represented by a node. The stream between two nodes is a downstream segment. Generally, a watershed is simply defined relative to a pour point and includes all areas where, if a raindrop falls, it will drain through the pour point. Each stream segment is associated with a catchment area defined by those points from which raindrops that fall would directly enter the stream segment. The ecological value of each stream segment is taken to be an ecological index of performance weighted by the water surface area.

The spatial-temporal dynamic optimization model for the present ecological value of the water resources of a watershed can now be developed as follows: Given the total funds available for treatment and other exogenously determined factors in the study area, the problem is specified as a total ecological value maximization problem subject to dynamic constraints – an inter-temporal investment constraint and spatial water quality constraints – and other constraints imposed by other physical and behavioral aspects of the problem. Temporal dynamic elements are introduced in the modeling process through the timing of investments in site specific treatment systems. The level of treatment that a specific system produces in any period (t) can be considered a function of the cumulative investment in the system. Spatial dynamics are introduced by the spatial distribution of investments in treatment within the watershed and the interaction with exogenously determined pollutant inputs that determines water quality at all points.

2.1. Objective Function

The objective function is to maximize the present value of the total ecological index over all streams in the watershed over the planning horizon (TEI):

$$\underset{C_{i,t}}{Max}(TEI) = \underset{C_{i,t}}{Max}\sum_{t=0}^{T}\sum_{i=1}^{I}\frac{EI_{i,t}}{(1+r)^{t}}$$

where:

i = 1, 2, …, I is the index for stream segments;

t = 0, 1, …, T in the index for time periods in years;

r is the rate of time preference for ecological services;

$C_{i,t}$ is the investment in watershed remediation/water treatment in segment i during time t;

$EI_{i,t}$ is the value of the ecological index for segment i at time t.

Commonly used measures relevant to ecological services that could serve as a weight for the primary index include stream miles, stream area, stream order, and the maximal area of the watershed drained by the stream segment. The technical team for the Cheat project chose to use stream surface area, a continuous cardinal measure, to weight the ecological coefficient based on the observation that ecological productivity is roughly proportional to surface area. Potential ecological condition indices include species diversity, total biological productivity, targeted fish biomass, invertebrate based condition index, and fish based condition index. Ecologists working on the project have recommended the invertebrate based condition index, partially on scientific considerations and in part because this index is currently used in the Cheat by monitoring and regulatory agencies.

The ecological index for segment i at time t, $EI_{i,t}$, is the product of the stream surface area in segment i, SA_i, and the stream's ecological condition in segment i at time t, $EC_{i,t}(a_{i,t})$, which depends on water quality or pollutant concentration, $a_{i,t}$. That is:

$$EI_{i,t} = SA_i EC_{i,t}(a_{i,t})$$

where $a_{i,t} = \frac{y_{i,t}}{wf_{i,t}}$, $y_{i,t}$ is pollution loading in segment i during time t, and $wf_{i,t}$ is water flow. $EC_{i,t}(a_{i,t})$ is modeled as a step function to reflect ecologically based threshold responses of aquatic populations to changes in pollutant concentration. That is:

$$\begin{aligned} EC_{i,t}(a_{i,t}) &= e_1 \quad if \quad 0 \le a_{i,t} < A_1 \\ EC_{i,t}(a_{i,t}) &= e_2 \quad if \quad A_1 \le a_{i,t} < A_2 \\ &\vdots \\ EC_{i,t}(a_{i,t}) &= e_k \quad if \quad A_{k-1} \le a_{i,t} \end{aligned}$$

where $e_1, e_2, ..., e_k$ are the ecological values associated with each step or level of ecological services and $A_1, A_2, ..., A_{k-1}$ are the pollutant concentrations corresponding to the threshold levels that separate the k steps.

2.2. Constraints

Numerous factors are included via sets of constraints including the level of treatment as a function of total investment in water quality improvement projects, inter-temporal equations of motion which depend on the level of investment in treatment in each segment, spatial equations of motion which correspond to the imposition of a mass balance water quality model, and exogenously determined investment constraints.

(1) Treatment constraints are:

$$u_{i,t} = u_i CC_{i,t}$$

where:

u_i is the technical coefficient that maps effective capital investment in treatment systems into the realized treatment of each segment;

$u_{i,t}$ is the level of treatment in segment i during period t and is directly proportional to the cumulative investment (costs) $CC_{i,t}$ (i.e., the effective capital investments defined as the effectiveness of all investments through all t years within segment i).

(2) Intertemporal equations of motion are:

$$CC_{i,t} = \frac{CC_{i,t-1}}{(1+\delta)} + C_{i,t} = \sum_{\tau=0}^{t-1} \frac{C_{i,t-\tau}}{(1+\delta)^{\tau}}$$

where δ is degradation rate of the treatment which reflects the physical depreciation of the quality of the investment from year to year or can be presented as a function of all present and past investments.

(3) Spatial equations of motion are:

$$y_{i,t} = (\sum_{l\in\{i\}^{upstream}} y_{l,t}) + x_{i,t} - u_{i,t} \text{ for downstream segments}$$

where:

$\{i\}^{upstream}$ represents the set of segments directly upstream of segment i (i.e., those segments that flow directly into segment i);

$y_{i,t}$ is pollution loadings and can be equivalently given as $y_{i,t} = a_{i,t} wf_{i,t}$ (the product of pollution concentration, $a_{i,t}$, and water flow, $wf_{i,t}$) within each segment during each time period;

$x_{i,t}$ is the exogenously determined pollution load generated within the drainage area of segment i during period t.

The above equation represents a mass-balance approach. For headwater streams (i.e., those streams in the upper end of a watershed that only include direct flow), this reduces to:

$$y_{i,t} = x_{i,t} - u_{i,t} \text{ for headwater stream segments}$$

Note that pollution loadings in headwater streams are from sources within the catchments while loadings in downstream segments include the contribution from streams flowing in plus the contribution from the associated catchments (Ali, 2002). The level of treatment determines any reductions in pollution.

(4) Investment constraints are:

$$\sum_{i=1}^{I} C_{i,t} \le C_t^{\max}\ ,\ C_{i,t} \ge 0 \qquad \forall i,t$$

where $C_t^{\max}$ is the maximum level of investment for water quality projects available during time period t which can be divided among segments but investment is any segment is non-negative.

Assuming that the mass balance model is a reasonable approximation to a true water quality model and that sufficient information is available on concentrations and flow to calculate loadings in each segment during the base period, the exogenous loadings can be calculated by:

$$\overline{x_{i,0}} = \overline{y_{i,0}} - \sum_{l \in \{i\}^{upstream}} \overline{y_{l,0}} \text{ for downstream segments, and}$$

$$\overline{x_{i,0}} = \overline{y_{i,0}} \text{ for headwater stream segments}$$

This defines exogenous pollution from the drainage to segment i from respective sub-watersheds at the initial time period. With this definition, a positive difference for a downstream segment would indicate that more pollution loadings are being added from the point and non-point sources which flow into that downstream segment. A negative difference would indicate that some loads are being neutralized by existing treatment facilities. A zero difference would indicate that mass-balance assumptions are satisfied at the initial state as well (Ali, 2002).

In the above general model, there are two vectors of state variables: pollution loadings in each segment during each time period, $y_{i,t}$, and the level of treatment in each segment during each period, $u_{i,t}$. There is a single vector of choice variables during each time period: the additional investment within each segment in the treatment, $C_{i,t}$. The level of treatment in each segment is defined by the cumulative treatment from current and past investment and can be thought of as primarily an intertemporal variable. The pollution loadings in each segment during each period can be considered a spatial variable – determined by the level of the intertemporal state and the spatial equations of motion.

2.3. Mathematical Solution: the Kuhn-Tucker Conditions

From the above section, we obtain the complete general model:

$$\underset{C_{i,t}}{Max}(TEI) = \underset{C_{i,t}}{Max} \sum_{t=0}^{T} \sum_{i=1}^{I} \frac{SA_iEC_{i,t}(a_{i,t})}{(1+r)^t} \qquad (1)$$

where:

$$a_{i,t} = \frac{y_{i,t}}{wf_{i,t}} \tag{2}$$

$$\begin{aligned} EC_{i,t}(a_{i,t}) &= e_1 \quad if \quad 0 \le a_{i,t} < A_1 \\ EC_{i,t}(a_{i,t}) &= e_2 \quad if \quad A_1 \le a_{i,t} < A_2 \\ &\vdots \\ EC_{i,t}(a_{i,t}) &= e_k \quad if \quad A_{k-1} \le a_{i,t} \end{aligned} \tag{3}$$

Subject to:

Treatment constraints:

$$u_{i,t} = u_i CC_{i,t} \tag{4}$$

Intertemporal equations of motion:

$$CC_{i,t} = \frac{CC_{i,t-1}}{(1+\delta)} + C_{i,t} = \sum_{\tau=0}^{t-1} \frac{C_{i,t-\tau}}{(1+\delta)^{\tau}} \tag{5}$$

Spatial equations of motion:

$$y_{i,t} = \Big(\sum_{l \in \{i\}^{upstream}} y_{l,t}\Big) + x_{i,t} - u_{i,t} \text{ for downstream segments} \tag{6}$$

$$y_{i,t} = x_{i,t} - u_{i,t} \text{ for headwater stream segments}$$

Investment constraints:

$$\sum_{i=1}^{I} C_{i,t} \le C_t^{\max}, \quad C_{i,t} \ge 0 \qquad \forall i,t \tag{7}$$

In addition, the following initial conditions hold:

$$\begin{aligned} \overline{x_{i,0}} &= \overline{y_{i,0}} - \sum_{l \in \{i\}^{upstream}} \overline{y_{l,0}} \quad \text{for downstream segments, and} \\ \overline{x_{i,0}} &= \overline{y_{i,0}} \quad \text{for headwater stream segments} \end{aligned} \tag{8}$$

The Lagrangian expression for the constrained optimization problem may be written as:

$$
\begin{aligned}
L = \sum_{t=1}^{T} \sum_{i=1}^{I} \frac{(SA_i EC_{i,t}(\frac{y_{i,t}}{wf_{i,t}}))}{(1+r)^t} \\
+ \lambda_{i,t}(x_{i,t} - u_i \sum_{\tau=0}^{t-1} \frac{C_{i,t-\tau}}{(1+\delta)^{\tau}} - y_{i,t} + \sum_{l \in \{i\}^{upstream}} y_{l,t}) + \eta_t (\sum_{i=1}^{I} C_t^{\max} - C_{i,t})
\end{aligned}
\qquad (9)
$$

The Kuhn-Tucker conditions are a series of necessary conditions that must be satisfied for an optimal solution. The Kuhn-Tucker conditions for a maximum for the Lagrangian given in equation (9) are:

$$
\begin{aligned}
\frac{\partial L}{\partial C_{i,t}} = -\lambda_{i,t} u_i - \eta_t \le 0 \\
\frac{\partial L}{\partial C_{i,t}} C_{i,t} = (-\lambda_{i,t} u_i - \eta_t) C_{i,t} = 0
\end{aligned}
\qquad (10)
$$

$$
\frac{\partial L}{\partial \lambda_{i,t}} = x_{i,t} - u_i \sum_{\tau=0}^{t-1} \frac{C_{i,t-\tau}}{(1+\delta)^{\tau}} - y_{i,t} + \sum_{l \in \{i\}^{upstream}} y_{l,t} = 0 \qquad (11)
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \eta_t} = \sum_{i=1}^{I} C_t^{\max} - C_{i,t} \ge 0 \\
\frac{\partial L}{\partial \eta_t} \eta_t = (\sum_{i=1}^{I} C_t^{\max} - C_{i,t}) \eta_t = 0
\end{aligned}
\qquad (12)
$$

$$
C_{i,t} \ge 0, \quad \eta_t \ge 0, \quad \lambda_{i,t}\text{: free} \qquad (13)
$$

These technical mathematical equations can be interpreted simply and provide straightforward implications for management decisions. Consider an interpretation of the model structure. Equation (1) represents the objective of the decision maker – to choose the best treatment strategy given the amount of resources anticipated to maximize the value of ecological services (TEI) generated within the Cheat River watershed. Equation (2) is a definition that says that ambient water quality, $a_{i,t}$, is given by the ratio of the acid load to the stream flow. The third set of equations, (3), defines the level of the ecological index ($EC_{i,t}(a_{i,t})$) that corresponds to a range of acidity for each segment during each time period. While clearly a simplification of reality, this approach reflects the ecological thresholds that occur across a range of water quality parameters. The fourth equation indicates, at least for passive systems, that the treatment produced (alkalinity) and provided ($u_{i,t}$) is proportional to the current level of effective investment in remediation projects within the direct drainage of each stream segment. Equation (5) indicates that the amount of alkalinity generated from each investment will decline over time as projects age. The effective investment at any time is the accumulation of past investments after accounting for any depreciation in output. The water quality within each segment is represented by equation (6) which accounts for the pollution added in each segment as well as treatment and includes that coming from all upstream segments after appropriate reductions for all treatment programs. The resource available for additional remediation programs each year is given in (7) where $C_t^{\max}$ is the amount of money available each year for additional investment. Equation (8) provides the initial conditions for the model where the amounts of treatment from any past activities or natural occurring alkalinity sources are calculated from source data or from the output of any available water quality model. Equation (9) represents the augmented equation for final solution. This indicates that an equivalent problem can be solved where the original objective is augmented by the constraints specified. This is a mathematical restatement and adds little to the understanding of the model.

The operational information is provided by the elements included as equations (10) – (13). First consider two variables, $\lambda_{i,t}$ and η_t. The first, $\lambda_{i,t}$, represents the increase in ecological services over the entire planning horizon that would be obtained by investing an additional dollar in remediation in the i^{th} segment during period *t*. The second, η_t, represents the increase in ecological services over the entire planning horizon that could be obtained by investing another dollar in remediation during time period *t*. While seemingly similar, they are significantly different. The first reflects the spatial effect of a particular segment and measures the ecological return to a dollar invested in a specified location. The second reflects the temporal effect of an investment at the most advantageous point in the watershed during a specified period. Equation (10) states that, at the optimum, the ecological return in each segment when an investment is made must equal the maximum possible for the watershed during that period. Equation (11) just says that the water quality model must be satisfied or imposed. Equation (12) is a simple statement that, in order to justify expenditures each year, there must be a positive benefit in terms of ecological services. Equation (12) is a restatement of the constraints on the choice variables.

Equation (10) is clearly the crux of the problem. A solution that satisfies this constraint will provide a spatially explicit solution for remediation investment throughout the planning horizon. This is just the information that a manager must have to make appropriate investment decisions. The empirical solution to these equations is developed using the GAMS computer software package (Hansen, 2004).

The Kuhn-Tucker conditions deal with the necessary conditions for this maximum problem. The existence of a maximum for the ecological index welfare function is determined by the sufficient conditions. For a maximum problem, the sufficient condition for the existence of a global maximum is that the objective function is quasi-concave, or the second-order conditions are non-positive for a single variable problem or negative definite for a vector of variables problem; the sets of constraints must define a feasible region which is everywhere convex.

3. The Specific Model for AMD Treatment in the Cheat River Watershed

3.1. The Study Area and Data Sources

The Cheat River flows north through West Virginia to the Monongahela River near the Pennsylvania border. The majority of the 1,435 square miles drainage area is located in northeastern West Virginia (Hansen, 2002; U.S. Environmental Protection Agency, 2000). This watershed consists of forests, farm lands, and coal sites.

As nonpoint and non-permitted sources, abandoned mine lands (AMLs) (lands impacted by surface and deep mining operations completed prior to the 1977 Surface Mining Control and Reclamation Act regulations (SMCRA)) contribute acid mine drainage significantly to the Cheat River watershed. Another important nonpoint AMD source is bond forfeiture sites (BFSs) (mines abandoned since 1977 SMCRA). Active mining operations as permitted mining point sources (there are 128 active mining discharge permits for the watershed) contribute little AMD as well.

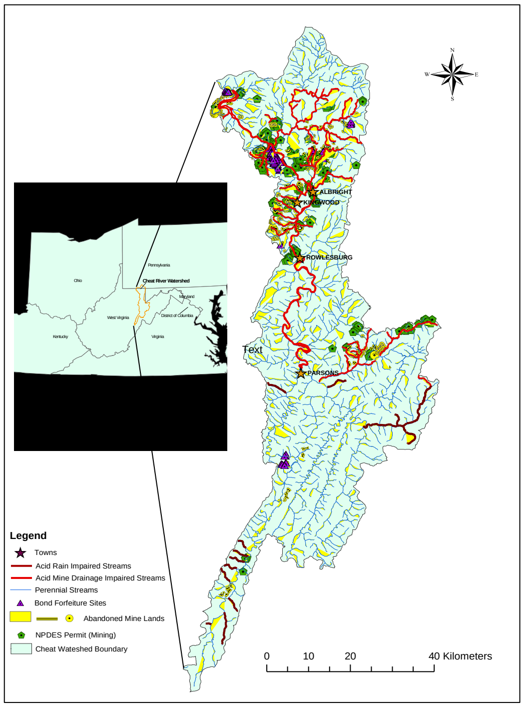
AMD is produced when water, oxygen, and a small amount of bacteria come into contact with pyrite, a mineral associated with coal in the Cheat River watershed. AMD is very acidic (low pH) water with high concentrations of metals such as iron, aluminum, and manganese. AMD pollutes streams, harms aquatic life including insects, reduces recreational activities, and destroys stream aesthetics due to orange discoloration from iron deposits. Today, many miles of streams within the Cheat River watershed fail to meet designated use, the result of over a century of mining (Friends of the Cheat, 2003).

Impaired by low pH and high levels of metals including iron, aluminum, manganese, and zinc, part of the main stem of the Cheat River and 54 other stream segments in the watershed were included in West Virginia’s 1998 303(d) list under the Aquatic Life and the Human Health use designation categories. Among these 55 polluted water bodies, one water body is zinc impaired, 50 water bodies have a low pH, 53 water bodies have iron and manganese impairments, and 55 water bodies have high aluminum content (U.S. Environmental Protection Agency, 2000).

The traditional approach to AMD relies on active treatment that uses alkaline chemical reagents and a mechanical system to neutralize the acidity. Recently, a variety of passive treatment systems such as open limestone channel and limestone leach beds have been developed to treat AMD with a low cost and little maintenance (Skousen, and Ziemkiewicz, 1996). Within the Cheat River watershed, earlier work by the River of Promise (i.e. ROP, which is a shared commitment for the restoration of the Cheat River; signatories included government agencies, environmental groups, and a coal company) focused on passive systems to fix AMD problems in the Big Sandy sub-basin and has proven successful. Application of passive systems to other AMD impaired streams is strongly recommended. In this specific model, AMD is assumed treated by open limestone channels. Figure 1 shows impaired and non-impaired stream segments, abandoned mine lands, bond forfeiture sites, and permitted mining points.

All data on water flow and stream area are average annual data, and calculated from the GIS-based data set based on the National Hydrography Dataset (NHD) which combines the US Geological Survey (USGS) Digital Line Graph (DLG) hydrography data with reach-related information from the USEPA Reach File Version 3. The data on net acidity are from West Virginia Division of Environmental Protection (WVDEP) and the Forestry Department at WVU (Petty and Barker, 2004). All data are 1:100,000-scale. The stream network of the Cheat River watershed is also given by GIS-based data set. Information on permitted mining sites, abandoned mine lands, and bond forfeiture sites is from the Natural Resource Analysis Center (NRAC) at West Virginia University (WVU).

Figure 1 Stream Segments in the Cheat River Watershed



3.2 The Specific Model

The objective function is to maximize the present value of the total ecological index for the Cheat River watershed (TEI):

$$Max(TEI) = \underset{\{c_{i,t}\}}{Max} \sum_{t=0}^{T} \sum_{i=1}^{I} \frac{EI_{i,t}}{(1+r)^t}$$

where:

i = 1, 2, …, 1793, the number of stream segments in National Hydrography Dataset (NHD) 1:100,000-scale coverage of the Cheat River watershed;

t = 0, 1, …, 10, the planning horizon in years.

The choice of the time preference, r, is controversial. A high r lowers the weight of ecological values received in the future which leads to the argument that discounting discriminates against future generations. Recent studies have utilized a value for r ranging from 3% (real rate of interest) to 7% (estimate of the opportunity cost of private capital by the Office of Management and Budget (OMB)) (Fletcher, et al., 2001). For this example, a value of 6% is used based on sensitivity test of interest rate.

The ecological index function depends on the pollution load which measures $a_{i,t}$ as net acidity in segment i at time t in mg/l. In the Cheat River watershed, $a_{i,t}$ is net acidity concentration which has the following properties:

$$\begin{array}{lll} a_{i,t} > 0 & if & pH < 7 \\ a_{i,t} = 0 & if & pH = 7 \\ a_{i,t} < 0 & if & pH > 7 \end{array}$$

From an ecological perspective, both excess alkalinity and acidity reduces ecological services. This is represented in the ecological condition function as:

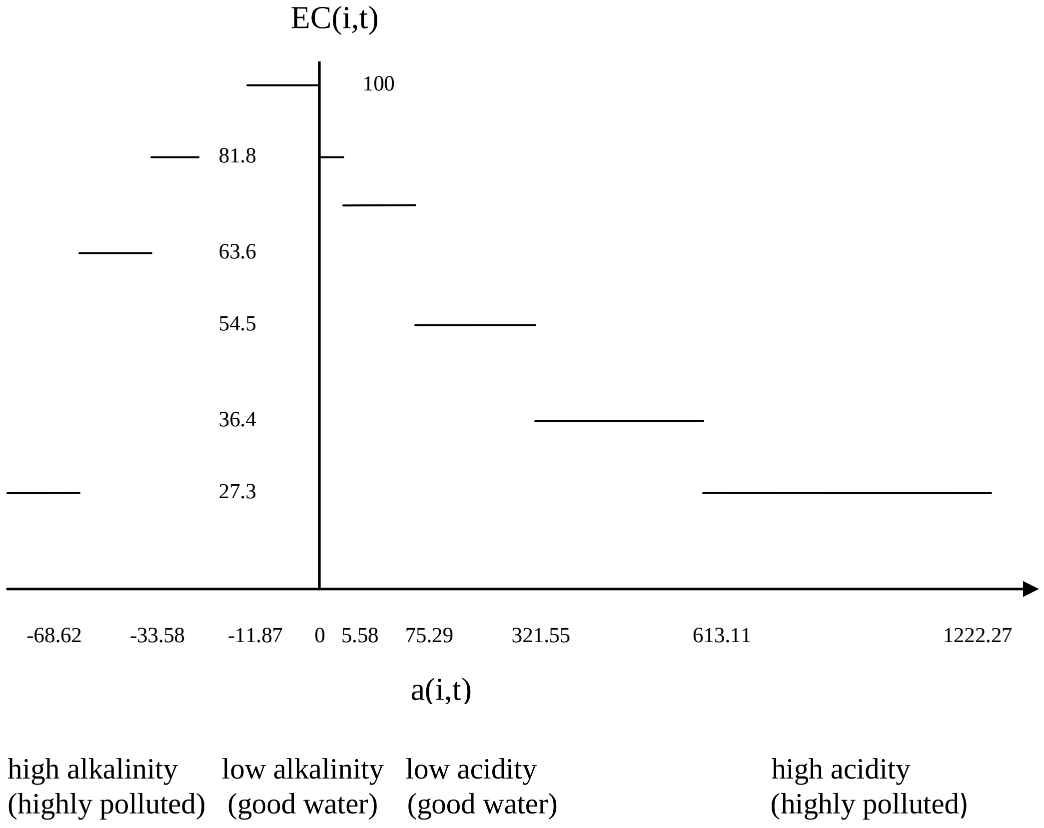
$$\begin{array}{lll} EC_{i,t}(a_{i,t}) = e_{-N} & if & a_{i,t} < A_{-(N-1)} \\ & \vdots & \\ EC_{i,t}(a_{i,t}) = e_{-2} & if & A_{-2} \le a_{i,t} < A_{-1} \\ EC_{i,t}(a_{i,t}) = e_{-1} & if & A_{-1} \le a_{i,t} < 0 \\ EC_{i,t}(a_{i,t}) = e_{1} & if & 0 \le a_{i,t} < A_{1} \\ EC_{i,t}(a_{i,t}) = e_{2} & if & A_{1} \le a_{i,t} < A_{2} \\ & \vdots & \\ EC_{i,t}(a_{i,t}) = e_{K} & if & A_{K-1} \le a_{i,t} \end{array}$$

where $e_1, e_2, ..., e_K$, $e_{-1}, e_{-2}, ..., e_{-N}$ are the ecological values associated with each step and $A_1, A_2, ..., A_{k-1}, A_{-1}, A_{-2}, ..., A_{-(N-1)}$ are net acidity concentrations corresponding to the threshold levels that separate the $K + N$ steps.

Specifically, based on analysis of available data, consider a ten-step ecological condition function for the Cheat River watershed as a function of net acidity represented by (Figure 2 is a graphical representation):

$$
\begin{aligned}
EC_{i,t}(a_{i,t}) &= 27.3 && \text{if} && a_{i,t} < -68.62 \\
EC_{i,t}(a_{i,t}) &= 63.6 && \text{if} && -68.62 \le a_{i,t} < -33.58 \\
EC_{i,t}(a_{i,t}) &= 81.8 && \text{if} && -33.58 \le a_{i,t} < -11.87 \\
EC_{i,t}(a_{i,t}) &= 100 && \text{if} && -11.87 \le a_{i,t} < 0 \\
EC_{i,t}(a_{i,t}) &= 81.8 && \text{if} && 0 \le a_{i,t} < 5.58 \\
EC_{i,t}(a_{i,t}) &= 72.7 && \text{if} && 5.58 \le a_{i,t} < 75.29 \\
EC_{i,t}(a_{i,t}) &= 54.5 && \text{if} && 75.29 \le a_{i,t} < 321.55 \\
EC_{i,t}(a_{i,t}) &= 36.4 && \text{if} && 321.55 \le a_{i,t} < 613.11 \\
EC_{i,t}(a_{i,t}) &= 27.3 && \text{if} && 613.11 \le a_{i,t} < 1222.27 \\
EC_{i,t}(a_{i,t}) &= 0 && \text{if} && 1222.27 \le a_{i,t}
\end{aligned}
$$

Figure 2 Estimated Step Ecological Condition Function in the Cheat River Watershed



(1) Treatment constraints are:

$$u_{i,t} = u_i CC_{i,t}$$

Since the early 1990s, a variety of passive treatment systems such as open limestone channels and limestone leach beds have been developed to treat AMD with low initial cost requiring little maintenance (Skousen, and Ziemkiewicz, 1996). The traditional approach to AMD relies on active treatment that uses alkaline chemical reagents and a mechanical system to neutralize the acidity. Within the Cheat River watershed, earlier work by the River of Promise (i.e. ROP, which is a shared commitment for the restoration of the Cheat River) focused on passive systems to fix AMD problems in the Big Sandy sub-basin and has proven successful. Application of passive systems to other AMD impaired streams is strongly recommended. In this paper, AMD is assumed treated by passive systems including open limestone channels and limestone leach beds. u_i is 0.006 for the mentioned passive systems in the Cheat River watershed since open limestone channels cost about $189 per ton of acid treated (Ziemkiewicz, 2003).

(2) Intertemporal equations of motion are:

$$CC_{i,t} = \frac{CC_{i,t-1}}{1+\delta} + C_{i,t}$$

In the Cheat River watershed, δ is assumed to be 0.02. Generally, alkalinity production is maximum at project initiation. Over time, the ability of a passive treatment system to generate alkalinity falls. δ represents the diminishing rate of alkalinity generation.

(3) Spatial equations of motion are:

$$y_{i,t} = \sum_{l \in \{i\}^{upstream}} y_{l,t} + x_{i,t} - u_{i,t} \text{ for downstream segments}$$

$$y_{i,t} = x_{i,t} - u_{i,t} \text{ for headwater stream segments}$$

Exogenously determined AMD generation for each segment during each period, $x_{i,t}$, is slowly decreasing over time. Peak acid load occurs sometime after mining, followed by a gradual decline over 20 or more years. To reflect this, AMD generated by abandoned mines is assumed to decrease over time at the rate α . Since many of the mining sites in the Cheat River watershed have a long history (over 50 years), a relatively low value for α (0.02) is used here based on sensitivity analysis of α . That is,

$$x_{i,t} = \frac{x_{i,t-1}}{1+\alpha} \text{ with initial conditions } x_{i,0} = \overline{x_{i,0}} \quad \forall \text{ segments } i .$$

$y_{i,t}$ is the annual acid load in segment i at time t. Annual acid load is the product of net acidity concentration ($a_{i,t}$) and water flow ($wf_{i,t}$). Average water flow data is in cubic-feet per second (cfs) and average net acidity is in milligrams per liter (mg/l). For water flow, the conversion factor is 448.84 when converting from cfs to gallons per minute (gpm). Annual acid loadings in metric tons of acid per year are the product of net acidity in gpm, water flow in mg/l, and a conversion factor 0.0019. In this model, the average water flow in each segment is used and is thus constant over time so that $y_{i,t} = a_{i,t} wf_i$.

(4) Investment constraints are:

$$\sum_{i=1}^{I} C_{i,t} \le C_t^{\max}$$

where:

$C_{i,t}$ is nonnegative, i.e. $C_{i,t} \ge 0 \quad \forall i, t$.

Finally, the initial loadings are given by:

$$\overline{x_{i,0}} = \overline{y_{i,0}} - \sum_{l \in \{i\}^{upstream}} \overline{y_{l,0}} \text{ for downstream segments, and}$$

$$\overline{x_{i,0}} = \overline{y_{i,0}} \text{ for headwater stream segments}$$

That is, given measured pollution loadings in each segment at time period 0, $\overline{y_{i,0}}$, the AMD generation to each segment at time period 0, $\overline{x_{i,0}}$, can be estimated. Then, assuming that AMD generation declines at the annual rate α, one notes:

$$x_{i,t} = \frac{x_{i,t-1}}{1+\alpha} \text{ with initial conditions } x_{i,0} = \overline{x_{i,0}} \quad \forall \text{ segments } i$$

any time period, t = 1, 2, …, 10.

3.3. Computer Solution to the Specific Model

The specific model can be solved using mixed integer programming (MIP) based on the assumptions about the forms of the objective function and constraints (Ali, 2002). To represent the step ecological condition function, binary variables $b_{i,t,k}$ are introduced so that the ecological condition function is written as follows:

$$\sum_{t=1}^{T} \sum_{i=1}^{I} \frac{(SA_i)(EC_{i,t}(a_{i,t}))}{(1+r)^t} = \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{k=1}^{K} \frac{(SA_i)(EC_{i,t,k})(b_{i,t,k})}{(1+r)^t}$$

$$\sum_{k=1}^{K} b_{i,t,k} = 1 \qquad \forall t,i$$

$$a_{i,t} \le \sum_{k=1}^{K} (m-(b_{i,t,k})(m-A_k))$$

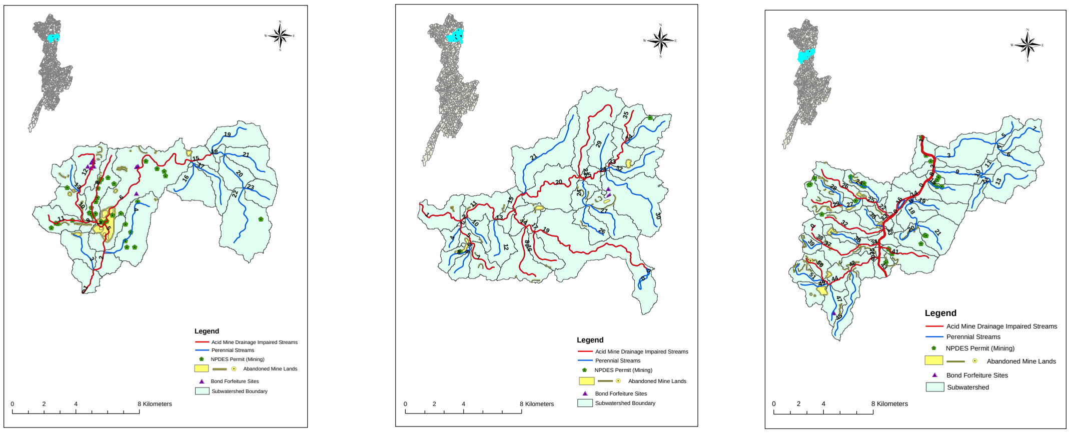
where m is a large number (1,000,000 here) and $k = 1,2,...,K$ are the numbers of the steps in the ecological condition function (Note: when $k = K$ then $A_k = m$). The above three constraints force a single, maximal value for $EC_{i,t,k}$ to be chosen to assure consistency with the step function.

CPLEX solver in GAMS was chosen to run the empirical model. The solution of integer programs with GAMS is achieved basically by introducing a new class of variable declaration statements and by invoking an IP solver. The declaration statement identifies selected variables to either be BINARY (zero one) or INTEGER.

4. The Empirical Results of Three Selected Subwatersheds

Based on their availability of data, degree of pollution, and potential for water quality trading, three subwatersheds – the Muddy Creek subwatershed, Little Sandy Creek subwatershed, and Albright Region subwatershed are selected to conduct the empirical analyses. Figure 3 illustrates these three subwatersheds.

Figure 3 The Muddy Creek, Little Sandy Creek, and Albright Region Subwatershed



There are many nonpoint sources – AMLs and BFSs, and point sources – active operations in both the Muddy Creek subwatershed and Albright Region subwatershed. Both subwatersheds are severely impaired by AMD. A part of the main stem of the Cheat River flows north through the Albright Region subwaterdshed. A coal-fired steam electric generating plant operates along the Cheat River in Albright and emits thermal effluent into the Cheat River. In the Little Sandy Creek subwatershed, there are many nonpoint sources – AMLs and BFSs, and a few point sources – active operations. This subwatershed is moderately impaired by AMD.

For the Muddy Creek subwatershed, I =23, T =10, α =0.02, δ =0.02, and $C_t^{\max}$ =\$50,000. For the Little Sandy Creek subwatershed, I =35, T =10, α =0.02, δ =0.02, and $C_t^{\max}$ =\$600. For the Albright Region subwatershed, I =51, T =10, α =0.02, δ =0.02, and $C_t^{\max}$ =\$50,000.

All GAMS results for these three subwatersheds show that (1) the spatial and temporal distribution of annual AMD treatment investments; (2) the spatial and temporal distribution of level of AMD treatment (acid loading reductions); (3) the spatial and temporal distribution of water quality (acid loadings and acid concentration); (4) the spatial and temporal distribution of ecological condition and ecological index; and (5) the objective value.

The objective values are 177,658,220, 371,702,686, and 1,380,298,081 ecological units respectively for the Muddy Creek subwatershed, Little Sandy Creek subwatershed, and Albright Region subwatershed. The Albright Region subwatershed has the biggest objective value mostly due to its largest amount of stream segments and great yearly maximum investment over all segments.

Since investments over time and space are important information and provide useful insight for stakeholders, we take the Muddy Creek subwatershed as an example to examine its optimal spatial and temporal investment distribution and AMD treatment distribution.

Table 1 The Spatial and Temporal Distribution of AMD Treatment Investments (C) in the Muddy Creek Subwatershed (Base Case) (Thousands of Dollars)

	Time Period									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Stream										
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	16.29	0.00	50.00	34.56	0.00	31.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	50.00	0.00	15.44	50.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19.00	50.00
12	6.83	50.00	50.00	33.71	0.00	0.00	0.00	0.00	0.00	0.00
13	36.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	3.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	2.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Several observations can be made based on Table 1. We see that: i) the investments in AMD treatment are distributed among severely impaired stream 8 and moderately impaired streams 10, 11, 12, 13, 19, and 21 over the 10-year planning horizon. For example, the investments in stream 8 in the 4th, 6th, 7th, and 9th years are 16.29, 50.00, 34.56, and 31.00 thousand dollars respectively as stream 8 is severely impaired by AMD with pH value of 2.80; and the investment in stream 19 in the 1st year is 3.87 thousand dollars as stream 19 is moderately impaired by AMD with pH value of 4.60; ii) not all severely or moderately impaired streams are given remediation in any time period. These exceptions are: stream 7 that is severely impaired but no investment is allocated, and streams 1, 2, 5, and 9 that are moderately impaired but no investments are distributed due to the restriction of maximum yearly investment over all stream segments that is only \$50,000; and iii) every year, the sum of the investments allocated in targeted stream segments is \$50,000. Take the investments in the 4th year as an example. The investments distributed in streams 8 and 12 are 16.29 and 33.71 thousand dollars separately, and the sum of these two investments is exactly \$50,000.

Corresponding distribution of AMD treatment is shown in Appendix 1: although investments are allocated in a certain time period, the treatment occurs not only in the treatment period but also in the time periods after since the model is a temporal model represented by intertemporal equations of motion. Take stream 13 as an example: investment is allocated only in the 1st year at 36.90 thousand dollars, but the treatment occurs not only in the 1st year but also in all years after (from the 2nd year to the 10th year) that are 221, 217, 213, 209, 205, 201, 197, 193, 189, and 185 tons of acid per year (tpy) respectively. The amount of AMD treatment decreases over time due to the degradation of the treatment that reflects the physical depreciation of the quality of the investment from year to year.

In general, the empirical results for the Muddy Creek, Little Sandy Creek, and Albright Region subwatersheds are very similar and support similar conclusions: the investments in AMD treatment are distributed among severely impaired stream and moderately impaired streams; but not all severely or moderately impaired streams are given remediation in any time period; and the treatment occurs not only in the treatment period but also in the time periods after.

The primary parameters in the general model are the same for every empirical model except for the number of stream segments, I , (23 for the Muddy Creek, 35 for the Little Sandy Creek, and 51 for the Albright Region subwatershed) and the maximum investments for remediation projects available each year, $C_t^{\max}$, (\$50,000 for the Muddy Creek and the Albright Region subwatersheds and \$600 for the Little Sandy Creek subwatershed).

Investment limits of \$50,000 and \$600 as $C_t^{\max}$ for the Muddy Creek and Little Sandy Creek subwatershed were chosen to represent reasonable results. If the number is smaller, fewer polluted streams may be treated, treatment will occur at lower levels with less investment, and total ecological index decreases. As the number decreases, fewer impaired streams are treated. On the other hand, as the available investment increases, more impaired or polluted streams are treated with greater investments or there are additional improvements of water quality in the same streams, thus the total ecological index increases. However, no significant room is left for trading to occur to further improve water quality in polluted streams.

5. Conclusions

The model developed in this paper is a spatial and temporal dynamic optimization model for water quality trading within a watershed framework. Temporal dynamic elements are introduced in the modeling process through the timing of investments in the treatment systems. Spatial dynamics are introduced by the spatial specification of each investment decision within the watershed and interaction of exogenously determined pollutant inputs with treatment. The system is constrained by the amount of resources available for investment in treatment systems within any time period. This model allows stakeholders to make informed decisions based on knowledge of complex spatial and temporal dynamics within a watershed.

The structure of the model is derived from a GIS-based water quality model. The GAMS/Cplex mixed-integer programming package can solve the specific model and give us the spatial and temporal distribution of AMD treatment investments, water quality, and ecological value. The empirical results from three subwatersheds suggest that available investments should be concentrated in heavily and moderately impaired stream segments.

The general model can be used to access the economic implications of alternative TMDL implementation strategies in different watersheds. Especially, the specific model can demonstrate the impacts of watershed-based AMD pollutant trading among sources across both space and time within the Cheat River watershed. The USEPA released a draft TMDL for AMD in the Cheat River watershed (U.S. Environmental Protection Agency, 2000) in 2000 and then helped West Virginia establish a statewide advisory stakeholder group. Under the TMDL process, the stakeholder committee developed a pollutant trading framework for AMD in the Cheat River watershed. In the Cheat River watershed, trading can potentially involve a variety of point and nonpoint sources. The specific model developed in this paper can be utilized to demonstrate the scope of pollution trading between and among permitted and abandoned mines across time. Similarly, the model could be used to evaluate the ecological implications of the AMD component of any proposed trade.

Appendix 1

The spatial and temporal distribution of the level of AMD treatment (u) in the Muddy Creek subwatershed (metric tons of $CaCO_3$ equivalent per year (tpy))

	Time Period									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Stream										
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	98	96	394	594	582	757	742
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	300	294	381	674	660	647
11	0	0	0	0	0	0	0	0	114	412
12	41	340	634	823	807	791	776	761	746	731
13	221	217	213	209	205	201	197	193	189	185
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0
19	23	23	22	22	21	21	21	20	20	19
20	0	0	0	0	0	0	0	0	0	0
21	14	14	14	14	13	13	13	13	12	12
22	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0

References

Ali, Md. Kamar. 2002. *Stream Water Quality Management: A Stochastic Mixed-integer Programming Model*, Unpublished Ph.D. Dissertation, West Virginia University, Morgantown, WV.

Baumol, W.J. and W.E. Oates. 1988. *The Theory of Environmental Policy*. 2nd ed. New York: Cambridge University Press.

Boyd, James. 2000. "The New Face of the Clean Water Act: A Critical Review of the EPA's Proposed TMDL Rules," *Discussion Paper 00-12*, Resources for the Future, March.

Brozovic, Nicholas, David L. Sunding, and David Zilberman. 2002. "Optimal management of Groundwater Over Space and Time," *In press*, in *Frontiers in Water Resource Economics*, ed. D. Berga and R. Goetz, Kluwer Academic Publishers.

Burnett, Kimberly M., Brooks A. Kaiser, and James A. Roumasset. 2007. "Invasive Species Control over Space and Time: Miconia calvescens n Oahu, Hawaii," *Journal of Agricultural and Applied Economics*. 39(October): 125-132.

Conrad, Jon M. and Colin W. Clark. 1987. *Natural Resource Economics: Notes and Problems*, Cambridge University Press.

Deal, Brian, Cheryl Farello, Mary Lancaster, Thomas Kompare, and Bruce Hannon. 1999. "A Dynamic Model of the Spatial Spread of an Infectious Disease: The Case of Fox Rabies in Illinois," [online], August 1999 [cited 18 May 2003]. Available from World Wide Web: (http://www.rehearsal.uiuc.edu/NSF/report/SFox-2.html)

Downstream Stratergies, LLC (Hansen, Evan, Martin Christ, Jerald Fletcher, J. Todd Petty, Paul Ziemkiewicz, and Richard S. Herd). 2004. *The Potential for Water Quality Trading to Help Implement the Cheat Watershed Acid Mine Drainage Total Maximum Daily Load in West Virginia*. Submitted to U.S. Environmental Protection Agency, Region 3, April.

Ejeta, Messele Zewdie. 2000. *Optimal Integrated Water Resources Planning in Watersheds with Limited Water Resources*, Unpublished Ph.D. Dissertation, Arizona State University.

Elder, Don, Gayle Killam and Paul Koberstein. 1999. The Clean Water Act: An Owner's Manual, Portland, OR: River Network.

Fletcher, Jerald J., Jack A. Fuller, and Tim T. Phipps. 2001. *Watershed Economics: A Guide to Watershed Management*, Prepared for the U.S. Department of Energy, National Energy Technology Laboratory, Morgantown, WV.

Friends of the Cheat. 2003. *Natural Qualities of the Cheat River* [online], 2003 [cited 18 May 2003]. Available from World Wide Web: (http://www.cheat.org/Cheat_River/cheat_river.html)

Funk III, Robert Edward. 1993. *Identifying a Cost-effective Solution for Treating Acid Mine Drainage in the Middle Fork River Watershed*, Unpublished M.S. Thesis, West Virginia University, Morgantown, WV.

Gaudet, Gerard, Michel Moreaux, and Stephen W. Salant. 1998. *Intertemporal and Spatial Depletion of Landfills,* Fondazione Eni Enrico Mattei Note di Lavoro publishing.

Greiner, Romy, and Oscar Cacho. 2001. "On the Efficient Use of a Catchment's Land and Water Resources: Dryland Salinization in Australia," *Ecological Economics* 38(3-September):441-458.

Hansen, Evan. 2002. "Cheat River Acid Mine Drainage TMDL Case Study: Increasing Stakeholder Confidence in computer Models," [online – cited 18 May 2003]. Available from World Wide Web: (http://www.wef.org/pdffiles/TMDL/Hansen.pdf)

Horan, Richard D. 2001. “Differences in Social and Public Risk Perceptions and Conflicting Impacts on Point/Nonpoint Trading Ratios,” *American Journal of Agricultural Economics*. 83(4):934-944.

Horan, Richard D., James S. Shortle. 2005. “When Two Wrongs Make a Right: Second-best Point-nonpoint Trading Ratios,” *American Journal of Agricultural Economics*. 87(2): 340-352.

Jagger, Thomas H., Xufeng Niu, and James B. Elsner. 2002. “A Space-Time Model for Seasonal Hurricane Prediction,” *International Journal of Climatology*, 22(March): 451-465.

Labys, Walter C., Takashi Takayama, and Noel D. Uri, Ed. 1989. “Computing Dynamic Spatial Oligopolistic Equilibrium in an Exhaustible Resource Market: The Case of Coal,” In *Quantitative methods for market-oriented economic analysis over space and time*, pp. 87-10. Aldershot, U.K.; Vt. Brookfield. Hong Kong and Sydney: Gower, Avebury.

Liebman, J.C. and W.R. Lynn. 1966. “The Optimal Allocation of Stream Dissolved Oxygen,” *Water Resources Research*. 2(3):581-91.

Malik, Arun S., David Letson, and Stephen R. Crutchfield. 1993. “Point/Nonpoint Source Trading of Pollution Abatement: Choosing the Right Trading Ratio,” *American Journal of Agricultural Economics*. 75(November):959-967.

Markris, Miltiadis. 2001. “Necessary Conditions for Infinite-Horizon Discounted Two-Stage Optimal Control Problems,” *Journal of Economic Dynamics and Control*. 25(12-December):1935-1950.

National Wildlife Federation. 1999. “A New Tool for Water Quality: Making Watershed-Based Trading Work for You,” *People and Nature: Our Future is in the Balance*, June.

Noel, Jay Everett. 1979. Dynamic Water Management: An Optimal Control Approach, Unpublished Ph.D. Dissertation, University of California, Davis, CA.

Opaluch, James Joseph. 1981. *River Basin Management: The Optimal Control of Water Quantity and Quality*, Unpublished Ph.D. Dissertation, University of California, Berkeley.

Pace, R. Kelley, et al. 2000. “A Method for Spatial-Temporal Forecasting with an Application to Real Estate Prices,” *International Journal of Forecasting.* 16(2-April-June):229-246.

Petty, T., and J Barker. 2004. Water Quality variability in Tributaries of the Cheat River, a mined Appalachian watershed. Proceedings of the 2004 American Society of Mining and Reclamation.

Renshaw, Eric. 1993. *Modeling Biological Populations in Space and Time*, Cambridge University Press: August.

Riddel, Mary. 2001. “A Dynamic Approach to Estimating Hedonic Prices for Environmental Goods: An Application to Open Space Purchase,” *Land Economics.* 77(4-November):494-512.

Shortle, James S. 1990. “The Allocative Efficiency Implications of Water Pollution Abatement Cost Comparisons,” *Water Resources Research*. 26(5):793-797.

Shortle, James S. and Richard D. Horan. 2001. “The Economics of Nonpoint Pollution Control,” *Journal of Economic Surveys*. 15(3):255-289.

Skousen, Jeffrey G., and Paul F. Ziemkiewicz. 1996. *Acid Mine Drainage: Control & Treatment*, Second edition, West Virginia University and the National Mine Land Reclamation Center, Morgantown, West Virginia.

Stephenson, Kurt, Patricia Norris, and Leonard Shabman. 1998. “Watershed-based Effluent Trading: The Nonpoint Source Challenge,” *Contemporary Economic Policy*. 16(October):412-421.

U.S. Environmental Protection Agency, Region 3. 2000. *Metals and pH TMDLs for the Cheat River Watershed*, West Virginia (Draft Report), December.

U.S. Environmental Protection Agency, Office of Water. 2002. *Water Quality Trading Policy Statement* (proposed), April.

West Virginia Division of Environmental Protection, Office of Water Resources, Watershed Assessment Program. 1996. *An Ecological Assessment of the Cheat River Watershed 1996*.

Zawack, Daniel J. and Gerald L. Thompson. 1983. *A Dynamic Space-Time Network Flow Model for City Traffic Congestion*, Pittsburgh, PA: Design Research Center CMU, December.

Zhao, Xiaobing and Jerald J. Fletcher. 2004. "An Optimal Control Approach to Water Quality Trading: Cost-Effective Pont/Nonpoint Management in a Watershed Framework," Presented at the AAEA Annual Meetings, Denver, Colorado, August 1-4.

Ziemkiewicz, Paul. 2003. "Proposed Strategy for Quantifying Pollution Trades," WVWRI (Draft), Presented at Cheat TMDL Stakeholder Group Meeting on March 5.

Ziemkiewicz, Paul, and Jennifer S. Simmons. 2003. "Completed and Future Projects on the Cheat River and Use of TMDL Trading," Presented at 2003 West Virginia Surface Mine Drainage Task Force Symposium, April.