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The *p*-mode spectrum of γ Equ (HR 8097)

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ABSTRACT

We present new high-speed photometric observations of the rapidly oscillating Ap (roAp) star γ Equ acquired on 26 nights during 1992. A frequency analysis of these observations reveals the presence of four *p* modes with frequencies of 1339, 1366, 1397 and 1427 μHz. These frequencies are all roughly equally spaced by about 30 μHz, which we interpret as the spacing of alternating even and odd ℓ modes for a slightly evolved A-type star. This interpretation leads to an asteroseismological luminosity estimate for γ Equ of $M_V(\Delta\nu) = 1.8$, which is in good agreement with the parallax luminosity $M_V(\pi) = 1.9$.

Key words: stars: chemically peculiar – stars: individual: HR 8097 – stars: oscillations.

1 INTRODUCTION

The rapidly oscillating Ap (roAp) stars are a class of cool, magnetic chemically peculiar A–F V stars which exhibit non-radial *p*-mode pulsations with periods in the range 6–16 min. The fact that the pulsations are seen in integrated light, and the shortness of the periods together imply that the pulsations are high-overtone ($n > 15$) modes of low degree ($\ell \leqslant 3$). The photometric amplitudes of the variations are observed to be in the range 0.07 mmag (an observational lower limit) to several mmag in Johnson *B* light. The pulsations of the roAp stars are thus similar to those seen in integrated sunlight but with amplitudes three to four orders of magnitude larger. Another difference between the pulsations in the Sun and the roAp stars is the manner in which the pulsation energy is distributed among the possible modes. The solar pulsation spectrum exhibits all the modes expected within a given frequency range, whereas in the roAp stars only a small number of possible modes are excited to observable amplitudes. The distribution of pulsation amplitudes among the different modes is also time dependent in some Ap stars. As a rule, roAp stars in which only a few (say, 1–3) modes are excited have more stable amplitude spectra than stars in which many different modes are excited. Examples of 'mode-stable' stars are HD 101065 (Martinez & Kurtz 1990) and HR 3831 (Kurtz, Kanaan & Martinez 1993) which have had the same mode structure for over 16 yr. The best example of a star with highly variable mode structure is HD 60435 (Matthews, Kurtz & Wehlau 1987) which has been observed to pulsate predominantly in so many different modes that it is not possible to talk about a 'principal pulsation frequency' for that star in the same way as is done for the other roAp stars. In this paper we show that γ Equ behaves in a very similar fashion to HD 60435.

Readers interested in a more detailed introduction to the roAp phenomenon are referred to the recent, readily available reviews by Kurtz (1990), Matthews (1991), and Shibahashi (1991). Kurtz and Matthews focus on observational developments while Shibahashi concentrates on theoretical developments.

2 γ EQU – THE STORY THUS FAR

2.1 General background

γ Equ (HR 8097, HD 201601) is a very sharp-lined F0p star, and was one of the first stars in which the Zeeman effect was observed by H.W. Babcock (Babcock & Cowling 1953). The record of magnetic measurements initiated by Babcock and continued by others reveals very gradual magnetic variations over the past 40 yr. In the late 1940s and early 1950s γ Equ had a magnetic field of +500 G (Babcock 1958) which slowly declined, passing through zero some time in the early 1970s (Bonsack & Pilachowski 1974). By 1988, the field had approached −1000 G (Mathys 1991). Using the data available up until 1974, Bonsack & Pilachowski determined a 72-yr magnetic period for γ Equ. Leroy et al. (1994) have presented polarization measurements strongly supporting a rotation period in excess of 70 yr. Fitting the mean longitudinal magnetic field data available up until the end of 1991 with a sinusoid, Leroy et al. determined a best-fitting period of 77 ± 10 yr and, from magnetic considerations, placed an upper limit of $T = 110$ yr on the period of the variations. For the 77-yr solution they determined the following set of parameters for the field strength at the positive magnetic pole, the inclination, and the magnetic obliquity, respectively: $B_\mathrm{p} = 5500$ G, $i = 150°$ and $\beta = 80°$.

Leroy et al.'s polarization measurements are also highly significant in another respect, namely that of establishing conclusively the nature of the magnetic variations. In contrast with the second-slowest magnetic variable, HD 9996 (period 23 yr), the proof that the magnetic variation in γ Equ results from the rotation of the star was not firmly established prior to these measurements*. An alternative interpretation of the magnetic variations was provided by Krause & Scholz (1981) who suggested a magnetic cycle, like the solar cycle, to explain the long-term variations in γ Equ.

The variation of polarization expected for an oblique rotator (Stibbs 1950; Wolff 1983) with a dipolar magnetic geometry has been calculated by Landolfi et al. (1993) who expressed these variations in diagnostic plots that are actually observed in some magnetic stars (Leroy, Landi Degl'Innocenti & Landolfi 1993). For γ Equ, Leroy et al. (1994) showed that their linear polarization data, together with all the available circular polarization data, can be consistently interpreted within the framework of the oblique rotator model, thus establishing that the magnetic variations are indeed caused by oblique rotation.

2.2 The *p*-mode pulsations

In 1981 Kurtz (1983) discovered rapid oscillations with a 12.44-min period ($\nu = 1339$ μHz) in γ Equ. The Johnson B amplitude of the pulsations in these discovery observations varied between 0.5 and 1.4 mmag from night to night (and possibly on time-scales shorter than a night) but Kurtz was unable to identify further modulating frequencies with confidence. Although intriguing, Kurtz's results were only preliminary. The 1339-μHz frequency identification suffered from a severe 1-d^{-1} alias ambiguity and required confirmation. Furthermore, the amplitude modulation noted on such short time-scales hinted at the presence of other pulsation modes being excited in γ Equ. By studying the night-to-night amplitude variations in his data Kurtz was able to make a tentative identification of a 38-d modulation period. This period is too long to be caused by beating with nearby pulsation modes and too short to be the signature of rotation (as we now know with certainty from the polarization work of Leroy et al. described above). Later work has not confirmed this periodicity and it is now considered to be insignificant. For completeness, we mention three other published photometric studies by Weiss (1983), Schneider et al. (1987) and Burnashev, Malanushenko & Polosukhina (1988). Weiss' study essentially confirmed Kurtz's discovery observations whilst the latter two did not achieve a sufficiently low noise level to detect the oscillations.

The presence of further pulsation modes was confirmed by Libbrecht (1988) who presented time series radial velocity observations of γ Equ acquired on four consecutive nights in 1987 using the coudé spectrograph with an iodine absorption cell on the Hale 5-m telescope. On combining the data from all four nights Libbrecht identified oscillations at 1366 μHz and 1427 μHz (the latter with a strong 1-d^{-1} alias ambiguity), but not the 1339-μHz oscillation reported by Kurtz in his discovery observations. In addition to revealing two new modes in this star, Libbrecht's observations also showed that the mode growth and decay times were shorter than the four-day span of his data†. Such short coherence times are not uncommon in roAp stars with rich *p*-mode spectra, as we discussed in the Introduction.

Comparing Libbrecht's radial velocity amplitude with Kurtz's broad-band intensity oscillations yields a radial velocity-to-light ratio of (42 m s^{-1})/(1.6 mmag)(peak-to-peak) = 26 km s^{-1} mag^{-1}. This is only an order of magnitude estimate since the radial velocity and intensity measurements were not simultaneous. Nevertheless, the measurement is in good agreement with the 59 ± 12 km s^{-1} mag^{-1} determined by Matthews et al. (1988) from simultaneous time series photometry and spectroscopy of HR 1217. For comparison, the radial velocity-to-light amplitude ratio for the Sun is $\sim$80 km s^{-1} mag^{-1} (Libbrecht 1988).

Several attempts have been made to confirm Libbrecht's radial velocity observations. Those by Odell & Kreidl (Schneider et al. 1987) using the Coudé Feed Telescope at Kitt Peak and by Schneider & Weiss (1989) using the Coudé Auxiliary Telescope at ESO failed primarily because these smaller telescopes did not achieve a sufficiently high signal-to-noise ratio. Using the larger Canada–France–Hawaii Telescope, Scott & Matthews (private communication) have confirmed Libbrecht's detection of radial velocity variations.‡

In the oblique pulsator model (Kurtz 1990) the pulsation and magnetic axes coincide. Thus both the observed magnetic field and the observed amplitude of a low-order zonal pulsation mode should vary in the same way with aspect as the star rotates. When the star is magnetically pole-on the pulsation amplitude is at a maximum and when the star is magnetically equator-on the pulsation amplitude is at a minimum. The pulsations in γ Equ were first observed in 1981 when the star was already $\sim 40°$ past negative crossover. Since the pulsation amplitude was expected to continue increasing through the 1980s and peak in the early 1990s, we decided to arrange a multi-site campaign in 1992, hoping to find that the oscillation amplitude had increased markedly in the decade following

* In addition to causing magnetic variations in Ap stars, rotation also causes spectroscopic and mean light photometric variations, and this provides an independent method of determining the rotation period. Although rotation periods shorter than 70 yr have been suggested in some photometric studies, the weight of evidence supports a very long rotation period for this star (cf. Catalano & Renson 1984; Hensberge, Manfroid & Sterken 1992).

† The 1366-μHz oscillation actually appears as two frequencies, 1365 and 1369 μHz, in the Fourier transform of Libbrecht's four-night data set. These peaks are too closely spaced to be consecutive overtones and are most likely Fourier artefacts of the changing amplitude of a single mode at around $\sim$ 1366 μHz.

‡ For completeness we mention the independent, unsuccessful searches for radial velocity variations by Bychkov (1987) and Zverko et al. (1989) using the 6-m telescope of the Special Astrophysical Observatory in Russia.

Table 1. Journal of the 1992 high-speed photometric observations of γ Equ.

HJD start 2440000+	HJD end 2440000+	Duration hr	N_{40}	Duty cycle	σ mmag	Telescope	Observer
8873.54381	8873.57004	0.63	51	0.90	1.7	ESO 1.0-m	WWW
8874.57176	8874.74928	4.26	294	0.77	1.9	ESO 1.0-m	WWW
8875.47821	8875.75315	6.60	483	0.81	1.9	ESO 1.0-m	WWW
8876.49002	8876.75211	6.29	421	0.74	1.6	ESO 1.0-m	WWW
8877.48406	8877.75363	6.47	487	0.84	1.2	ESO 1.0-m	WWW
8878.47987	8878.75657	6.64	503	0.84	1.9	ESO 1.0-m	WWW
8879.48079	8879.75034	6.47	493	0.85	1.7	ESO 1.0-m	WWW
8880.48307	8880.75107	6.43	497	0.86	1.2	ESO 1.0-m	WWW
8891.63707	8891.84458	4.98	417	0.93	2.5	LO 1.1-m	MJN/TJK
8892.60084	8892.85521	6.10	505	0.92	2.3	LO 1.1-m	MJN/TJK
8893.58108	8893.84923	6.44	529	0.91	2.3	LO 1.1-m	MJN/TJK
8894.56483	8894.66988	2.52	210	0.93	1.7	PBO 0.4-m	MJT/MJN
8895.26240	8895.33739	1.80	150	0.93	1.9	SAAO 0.5-m	GRR
8895.58475	8895.82050	5.66	475	0.93	2.4	LO 1.1-m	MJN
8896.26906	8896.35591	2.08	172	0.92	1.4	SAAO 0.5-m	GRR
8896.61124	8896.82433	5.11	433	0.94	2.1	LO 1.1-m	MJN
8897.58909	8897.75306	3.94	328	0.93	1.8	LO 1.1-m	MJN
8898.26864	8898.39235	2.97	241	0.90	1.4	SAAO 0.5-m	GRR
8898.66161	8898.80645	3.48	297	0.95	2.1	LO 1.1-m	MJN
8899.16125	8899.34119	4.32	355	0.91	2.2	OAO 0.8-m	NID/TND/DEM
8900.15010	8900.36241	5.10	402	0.88	1.9	OAO 0.8-m	NID/TND/DEM
8900.25464	8900.39120	3.28	275	0.93	1.8	SAAO 0.5-m	GRR
8900.58765	8900.65351	1.58	125	0.88	1.6	LO 1.1-m	MJN
8905.99910	8906.14557	3.52	226	0.71	2.2	PO 0.6-m	PC
8912.99153	8913.10794	2.79	183	0.73	2.7	PO 0.6-m	PC
8913.98080	8914.10836	3.06	226	0.82	2.9	PO 0.6-m	PB
Σ		112.52	8778				

Notes: ESO = European Southern Observatory, LO = Lowell Observatory, PBO = Pine Bluff Observatory, SAAO = South African Astronomical Observatory, OAO = Odessa Astronomical Observatory, PO = Perth Observatory

Kurtz's 1981 observations and that further pulsation modes could be identified and studied. In this paper we will show that, although our expectations of higher amplitudes were not met, we identified four pulsation modes which enable us to derive a luminosity estimate for this star.

3 NEW HIGH-SPEED PHOTOMETRIC OBSERVATIONS OF γ EQU

In this paper we present new high-speed photometric observations of γ Equ acquired on 26 nights spanning the 40-day period JD 244 8873–8913 in 1992. These observations were acquired using the 1-m telescope of the European Southern Observatory (ESO), the 0.5-m telescope of the South African Astronomical Observatory (SAAO), the 1.1-m telescope of the Lowell Observatory (LO), the 0.8-m telescope of the Mt Dushak-Erekdag station of the Odessa State University Astronomical Observatory (OAO), the 0.4-m telescope at Pine Bluff Observatory (PBO) in Wisconsin, and the 0.6-m telescope at Perth Observatory. The observations were obtained through Strömgren v filters using continuous 10- or 20-s integrations that were later binned to produce 40-s integrations§.

§ We are not certain that the filter used in the SAAO observations was a Strömgren v filter. It may have been Johnson B, the closest of the Johnson filters to Strömgren v.

The data were corrected for coincidence counting losses, sky background and atmospheric extinction, in that order. After these basic reductions the data still contained some residual sky transparency variations, usually on time-scales of $\sim \frac{1}{2}$ h or more. These transparency variations gave rise to a number of low-frequency peaks in the Fourier transforms of the light curves. An appropriate set of low-frequency sinusoids was then subtracted from each light curve (Martinez (1993) gives a detailed account of this procedure). As a rule, we subtracted only sinusoids in the frequency range 0–694 μHz, but on nights strongly affected by sky transparency variations we extended this range up to 1099 μHz. We then selected a subset of what we term 'usable' light curves for the frequency analysis based on visual inspection of all the light curves and their Fourier transforms. Table 1 is a journal of the light curves selected for further analysis. It lists the Heliocentric Julian Dates of the first and last observation, the duration of the observations in hours, the number of 40-s integrations, the duty cycle (or 'filling factor') of the light curve, the standard deviation σ of the observations with respect to the mean for the night and the telescope/observer. The quantity σ contains contributions due to sky transparency variations, scintillation, photon statistics (negligible for this $V = 4.7$ star), and the actual variations of the star. Fig. 1 shows the Fourier amplitude spectra of the light curves listed in Table 1. Each panel in the figure contains the amplitude spectrum of the light curve acquired on the Julian Date listed at the top of that panel. The last two characters in the title of each panel serve to associate that panel uniquely

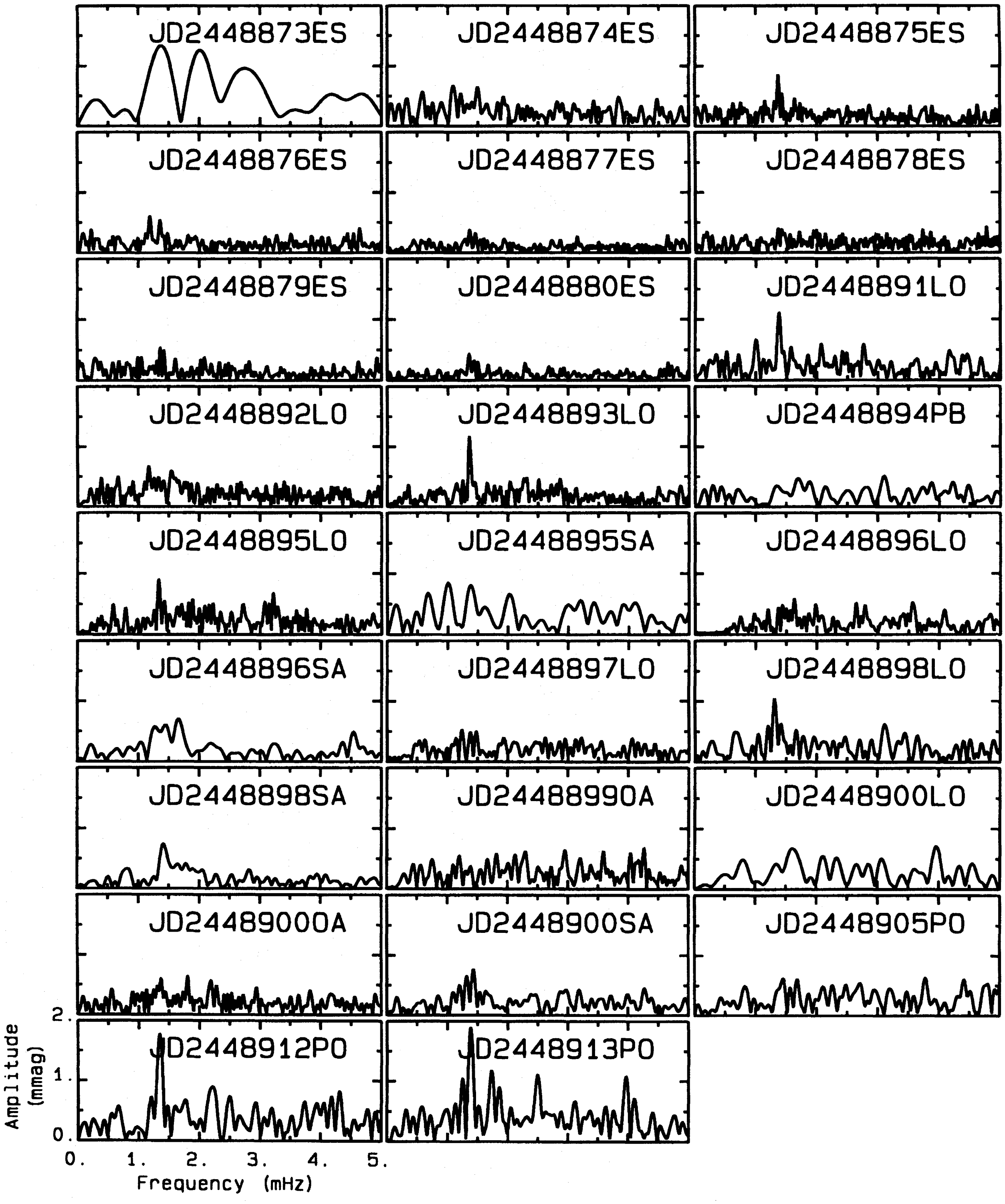

Figure 1. Amplitude spectra of the light curves listed in Table 1.

with an entry in Table 1 by identifying the observatory at which the light curve was acquired.

Photometric studies of the *p*-mode pulsations of γ Equ are difficult to perform owing to the very low pulsation amplitude of this star and the short mode life-times. Fig. 1 shows that in only about $\frac{1}{4}$ of the nights did γ Equ exhibit pulsations with an amplitude in excess of 1 mmag. Inspection of Fig. 1 and Table 1 shows that the ESO data are generally superior in quality to the rest of the data presented in this paper. The principal reason for this is that the noise in the ESO data is scintillation dominated and scintillation noise scales as the $-\frac{2}{3}$ power of the telescope aperture (Young 1967). Since the ESO data were acquired using a 1-m telescope, the largest used in this campaign, they have a correspondingly lower noise level.

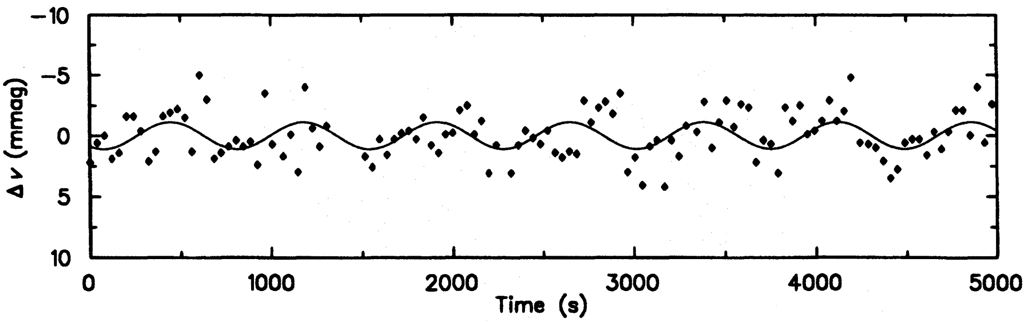

Figure 2. A portion of the Strömgren v light curve of γ Equ obtained at Lowell Observatory on night JD 244 8893. The solid line is the best-fitting sinusoid of frequency 1360 μHz. The Fourier transform of the full light curve is shown in Fig. 1.

Fig. 2 shows a portion of the light curve obtained at Lowell Observatory on night JD 244 8893. The solid line shows a least-squares fit of the best-fitting frequency (for that night) to the data.

4 FREQUENCY ANALYSES

The data were analysed for their component frequencies using Deeming's (1975) Discrete Fourier Transform (DFT) algorithm for unequally spaced data as coded by Kurtz (1985). The analysis was performed for individual nights and for groups of closely spaced nights. In what follows we present only the salient analyses.

The analyses presented in this paper make extensive use of prewhitening for the removal of frequencies from the data. By 'prewhitening' we mean that we identify a frequency in the DFT and optimize its amplitude and phase using linear least squares. That frequency, and its fitted amplitude and phase are then used as the starting values for an iterative non-linear least-squares fit which allows frequency, amplitude and phase to vary simultaneously and provides error estimates for all three parameters. The best-fitting sinusoid is then subtracted from the data in the time domain and the DFT of the residuals is computed to search for further frequencies. In this paper this two-stage fitting procedure is implicit in our use of the term 'prewhitening'.

In the roAp star literature it is customary to list amplitude and phase information for the frequencies identified in a given star. This paper marks a departure from this practice because, as we will show, the mode lifetimes in γ Equ are so short that no meaningful amplitude and phase information can be extracted. (Witness the night-to-night variation in Fig. 1.) Analyses of data combined from many nights is uninformative as the frequent mode changes lead to large numbers of spurious peaks in the DFTs of time series that span several to many mode life-times. Consequently, the more fruitful analyses are those of data spanning only several days.

To identify the frequency regime of interest we began by computing the DFT of the ESO JD 244 8873–8880 data in the domain 0–5000 μHz. Fig. 3 shows that the oscillations are confined to the narrow range of 1300–1500 μHz. In the analyses to follow we will examine the frequency range 1200–1700 μHz, a range wide enough to show the oscillations as well as a stretch of background noise on either side. Examining the region around 1400 μHz more closely, we see evidence for at least two component frequencies in this figure. Roughly a third of known roAp stars exhibit harmonics of their principal pulsation frequencies indicating that the pulsations are non-linear. Fig. 3 shows that the oscillations in γ Equ are not non-linear as no harmonics of the $\sim$ 1400-μHz oscillations are present above the noise level of 0.15 mmag.

4.1 The ESO JD 244 8873–8880 data

The JD 244 8873–8880 data taken at ESO comprise the largest (and best) data set acquired from a single site during the 1992 season. Fourier transforms of these data are shown in the top eight panels of Fig. 1. The amplitude spectrum of all of these nights together is shown in the top panel of Fig. 4. This figure shows two well-resolved window patterns standing out above the noise. The aliasing is too severe to permit an unambiguous identification of the frequencies. However, the tallest peaks in the two window patterns, 1365.29 $\pm$ 0.08 μHz and 1427.05 $\pm$ 0.13 μHz, correspond exactly to the two frequencies identified by Libbrecht (1988). This agreement suggests that both Libbrecht and we have identified the correct frequencies and not their 1-d^{-1} aliases. When these two frequencies are prewhitened the DFT of the residuals is essentially flat (Fig. 4, bottom panel). Analyses of the independent ESO subsets JD 244 8874–8876 and JD 244 8877–8880 confirm these findings, albeit at lower frequency resolution.

4.2 The Lowell JD 244 8891–8900 data

The JD 244 8891–8900 data acquired at Lowell Observatory form the second-largest of the 1992 data sets, but as a rule the light curves are shorter and have higher scatter than the ESO data. Consequently the analysis of the Lowell data is more complicated. A DFT of the entire Lowell data set (not shown) contains a mound of amplitude centred at around 1350 μHz but is otherwise uninformative. More meaningful results are obtained from the analysis of shorter subsets of the data.

The principal frequency in the JD 244 8891–8893 data suffers from a 1-d^{-1} alias ambiguity (Fig. 5 top); it is either 1356.7 $\pm$ 0.3 μHz or 1368.4 $\pm$ 0.3 μHz. Prewhitening the 1356.7-μHz peak yields a secondary frequency at 1385.5 $\pm$ 0.4 μHz (Fig. 5 middle). Prewhitening the 1368.4-μHz oscillation yields a secondary frequency at 1396.7 $\pm$ 0.4 μHz (Fig. 5 bottom). The 1427-μHz oscillation is absent above the 0.25-mmag noise level in these data.

A Fourier transform of the JD 244 8895–8900 Lowell data (Fig. 6) contains severely aliased window patterns that appear to be centred at around 1339 μHz (the single frequency

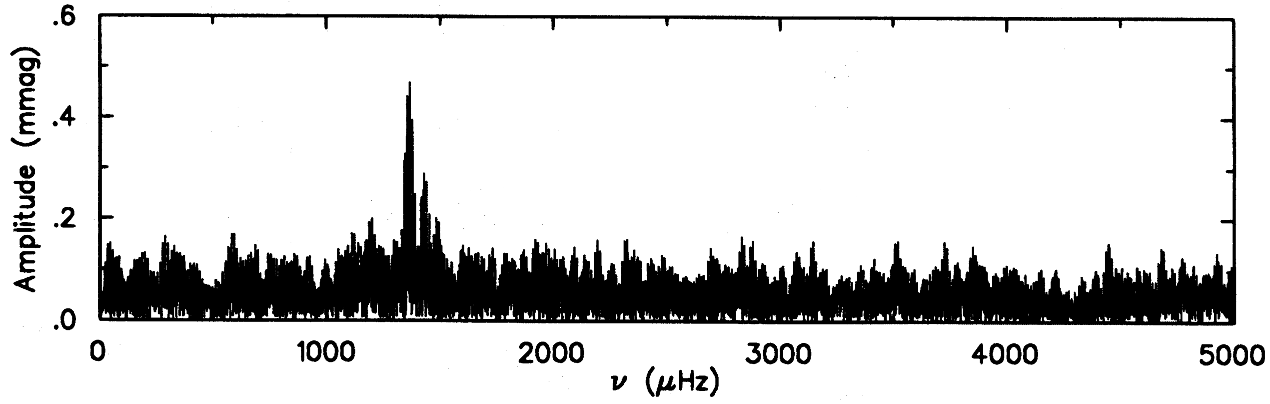

Figure 3. The amplitude spectrum of 1992 ESO data listed in Table 1.

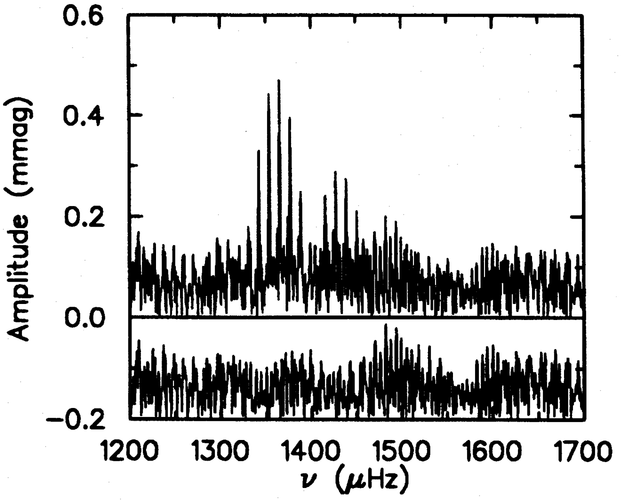

Figure 4. (Top) The amplitude spectrum of the ESO JD 244 8873–8880 data shows two resolved oscillations. (Bottom) The amplitude spectrum of the residuals after these oscillations are prewhitened.

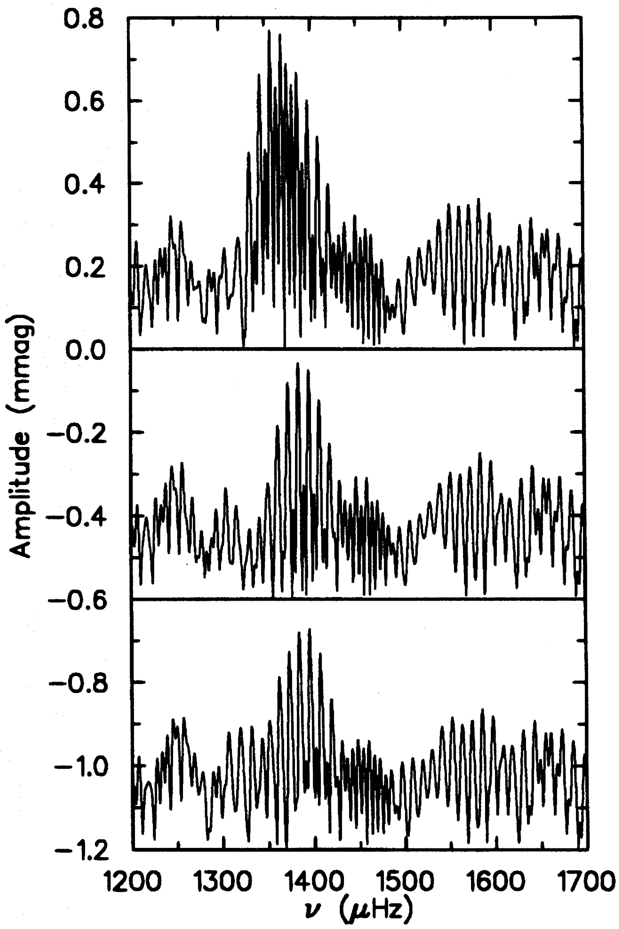

Figure 5. The amplitude spectrum of the JD 244 8891–8893 Lowell data (top), followed by the amplitude spectra of the residuals after prewhitening the two tallest peaks, 1356.7 μHz (middle) and 1368.4 μHz (bottom).

identified by Kurtz (1983)) as well as the 1427-μHz oscillations. Indeed, the tallest peak in Fig. 6 is at 1335.6 ± 0.2 μHz, and the tallest peak in the spectrum of the residuals on prewhitening this frequency lies at 1441.0 ± 0.3 μHz, the +1-d^{-1} alias of the 1427-μHz oscillation identified in the ESO data (Section 4.1) and by Libbrecht (1988).

It would be inappropriate to discuss only the features in Fig. 6 that agree with previous analyses whilst ignoring the marginally weaker features at ~ 1500 μHz and ~ 1650 μHz. These features are absent in other data sets and so weak in the data set under discussion here that we conclude that they are insignificant. The reason for presenting the analysis of this data set is to show in the next section that we find the same frequencies in independent contemporaneous data from another site. It is only this agreement among independent data sets that makes the results presented here credible.

4.3 The SAAO JD 244 8895–8900 data

The amplitude spectrum of the JD 244 8895–8900 data set (Fig. 7, top) contains two partially resolved window patterns. The tallest peak in the dominant window pattern is at 1427.1 ± 0.1 μHz. The aliasing is too severe for us to attach any confidence to this frequency in isolation. However, we note the excellent agreement with the results of Libbrecht (1988) and the new ESO data analysed above. Prewhitening the best-fitting sinusoid of frequency 1427.1 μHz allows us to isolate the window pattern of the secondary frequency in these data. The tallest peak in that severely aliased window pattern (Fig. 7, bottom) is at 1342.3 ± 0.2 μHz, which is close to the 1339-μHz oscillations identified by Kurtz (1983) and the 1335.6-μHz

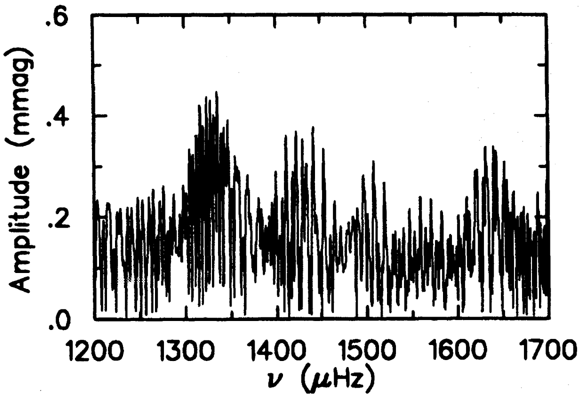

Figure 6. The amplitude spectrum of the Lowell JD 244 8895–8900 data suggests the presence of at least two well-resolved frequencies.

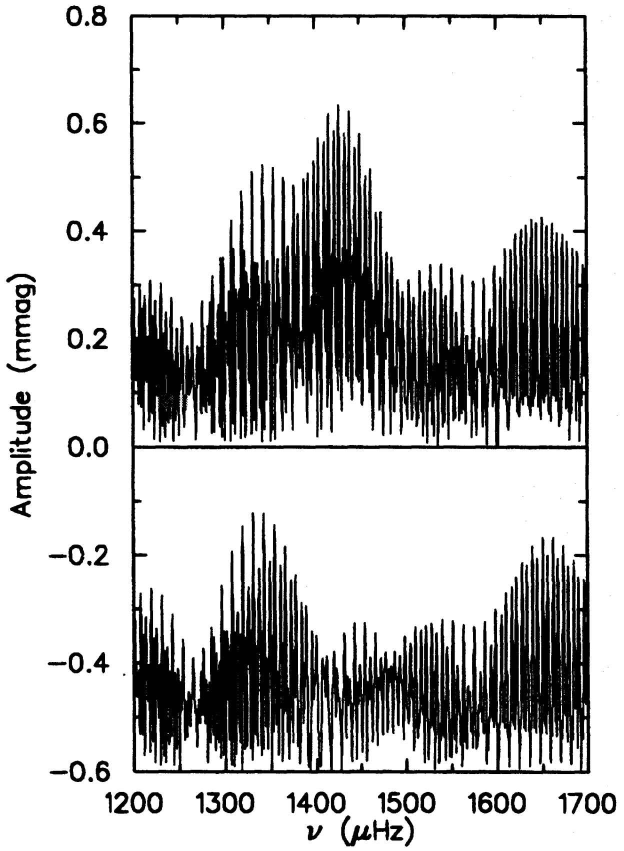

Figure 7. (Top) Amplitude spectrum of the SAAO JD 244 8895–8900 data showing a dominant oscillation at 1427 μHz. (Bottom) Prewhitening the dominant oscillation in the top panel reveals a secondary frequency at 1342 μHz.

oscillations identified in contemporaneous Lowell data. We did not combine the contemporaneous SAAO and LO data as we are not certain of which filter was used in the SAAO data.

4.4 The Perth Observatory and Odessa Observatory data sets

For completeness we mention results obtained from the two small data sets acquired at the Perth and Odessa Observatories. The JD 244 8912–8913 data acquired at Perth Observatory comprise only two rather short runs on consecutive nights. The window pattern of these data thus contains very strong daily aliases. Nevertheless, straightforward, unbiased prewhitening of the tallest peaks in the window patterns yields the frequencies 1365.3±0.4 μHz and 1426.6 ± 0.9 μHz.

The Odessa data comprise the two consecutive nights JD 244 8899 and 8900. The tallest peak in the severely aliased window pattern of the oscillations is at 1365.3 ± 0.8 μHz. No further information can be gained from this data set.

4.5 Reanalysis of Kurtz's 1981 data

Since the 1992 data provide only tentative confirmations of the 1339-μHz oscillation reported by Kurtz (1983), we decided to re-reduce and reanalyse Kurtz's 1981 data *ab initio.* Our somewhat more conservative reduction of those data yields the 1339-μHz oscillations identified by Kurtz (1983), but with a marginally cleaner spectral window and a smaller standard deviation of the residuals on fitting this frequency than that reported in the original analysis. In Kurtz's full data set the amplitude spectrum of the residuals on prewhitening the 1339-μHz oscillation does not indicate further signal content. However, it is possible to select a subset of his data (JD 244 4866–4869) in which the 1427-μHz oscillation is also present at a marginal level.

4.6 Reanalysis of Weiss' 1983 data

The first independent confirmation of Kurtz's discovery of rapid oscillations in γ Equ was published by Weiss (1983) based on data taken in 1983 with the 0.6-m University of Hawaii telescope on Mauna Kea. In light of the complexity of the oscillations in this star we decided to reanalyse those early observations in the hope of obtaining some additional information. By itself the analysis of these data is inconclusive although it does support the frequency identifications made earlier in this study. A straightforward, unbiased analysis yields two frequencies, 1320.9 μHz and 1389.6 μHz. However, the aliasing in these data is very strong and the $+1\text{-d}^{-1}$ aliases yield an equally plausible solution, 1332.6 μHz and 1401.2 μHz. The frequencies in this latter solution are in good agreement with those identified elsewhere in this paper.

5 DISCUSSION

The results of our various frequency analyses are summarized in Table 2. The table lists each data set analysed in Section 4 and the frequencies identified for that set. For comparison, the frequencies identified by Kurtz (1983) and Libbrecht (1988) are also listed, as well as the frequencies we found in Weiss' 1983 data. The column labelled 'Resolution' is a measure of the frequency resolution of the different data sets. It is the half-width at half-maximum of the central peak in the spectral window of a given data set. Frequencies with similar values have been grouped together in columns to facilitate our discussion. The easiest column to deal with is ν_4, the 1427-μHz oscillation. This oscillation appears in all the data sets presented in this paper as well as in Libbrecht's 1987 data. The 1441-μHz entry for the Lowell JD 244 8895–8900 data set is simply the $+1\text{-d}^{-1}$ alias of 1427 μHz. Next, we consider ν_2, where we see evidence

Table 2. Oscillation frequencies identified in γ Equ.

Data set JD 2440000+	ν_1 μHz	ν_2 μHz	ν_3 μHz	ν_4 μHz	Resolution μHz
Kurtz (1983)	1339				0.2
Weiss (1983)	1321		1390		
		or			2.6
	1333		1401		
Libbrecht (1988)		1366		1427	~ 2
8873-8880 ESO		1365		1427	1.0
8891-8893 LO		1357	1386		
		or			2.5
		1368	1397		
8895-8900 LO	1336			1441	1.6
8895-8900 SAAO	1342			1427	1.1
8912-8913 PO		1365		1427	3.9
8899-8900 OAO		1365			3.8

for oscillations at $\sim$1366 μHz in all but Kurtz's 1981 data. In the case of the Lowell JD 244 8891–8900 data, we have tabulated the two possible frequency solutions. If we select the solution with the 1368-μHz oscillation for ν_2, then there is another oscillation present at 1397 μHz, which we call ν_3. Turning our attention to ν_1, the clearest identification of the 1339-μHz oscillation to date remains that of Kurtz (1983). In this paper we have made tentative identifications of the 1339-μHz oscillation in the independent LO and SAAO data sets for JD 244 8895–8900. Our confidence in these marginal identifications is boosted by the fact that unbiased analyses of both data sets yield the same frequency solution. Lastly, we would like to draw readers' attention to the column labelled 'ν_3' which contains an ambiguous entry, 1386 μHz or 1397 μHz, identified in the Lowell JD 244 8891–8893 data (see Fig. 5). ¶ We believe the 1397-μHz solution to be the correct one although we do not have contemporaneous SAAO data to confirm this. In support of our belief, though, we note that 1397 μHz is well resolved from both the 1366- and 1427-μHz oscillations and is exactly mid-way between those frequencies. We summarize the above discussion by listing the frequencies identified in this study, viz. ν_1=1339 μHz, ν_2=1366 μHz, ν_3=1397 μHz and ν_4=1427 μHz. These frequencies are all roughly equally spaced by $\sim$ 30 μHz.

The asymptotic theory of low-degree, high-overtone ($n >> \ell$) p-mode pulsation (Tassoul 1990) predicts an eigenspectrum consisting of a comb of equally spaced frequencies $\nu_{n\ell}$ given by

$$\nu_{n\ell} = \Delta\nu(n + \ell/2 + \epsilon) + \text{ higher-order terms,} \quad (1)$$

where $\Delta\nu$ is the frequency spacing of consecutive overtones $(n, n+1, ...)$ and ϵ is a constant dependent on the equilibrium structure of the star. If modes of even and odd ℓ are excited, the observed frequency spacing is $\simeq \frac{1}{2}\Delta\nu$, as a simple inspection of equation (1) will show. The overtone spacing $\Delta\nu$ can be expressed in terms of structural parameters as $\Delta\nu \propto \sqrt{(GM/R^3)}$ and one may readily show, using the mass–luminosity relation, that the loci of constant $\Delta\nu$ are essentially lines of constant R in a theoretical Hertzsprung–Russell diagram. These loci have been calculated by Gabriel et al. (1985), Shibahashi & Saio (1985) and Heller & Kawaler (1988). The models all predict that $\Delta\nu \sim$ 50–80 μHz for slightly evolved A-type stars.

With the advent of Kurucz's (1993) metal-rich ATLAS9 model atmospheres codes, which contain much improved representations of line opacities over previous models, it has been possible to derive improved estimates of effective temperature and surface gravity for a number of Ap stars. For γ Equ, Adelman et al. (1995) obtain the best fit to optical spectrophotometric data and Hγ line profiles for a +0.5-dex metal enhancement model with $T_{\rm eff}$ = 7700 K and $\log g$ = 4.2. The A-star models mentioned above can be used to derive a luminosity estimate for this star from the observed frequency spacing. If we use Heller & Kawaler's (1988) models in particular and take $\Delta\nu$ = 60 μHz and $T_{\rm eff}$ = 7700 K then we determine an asteroseismological luminosity of $M_V(\Delta\nu) = 1.8$ for γ Equ. How does this compare with more direct luminosity estimates? The Bright Star Catalogue (Hoffleit 1982) gives a trigonometric parallax of π = 0.028 arcsec (about 36 pc) for γ Equ. Combining this with the apparent visual magnitude of $V = 4.7$ yields a parallax luminosity of $M_V(\pi) = 1.9$ for this star. The agreement between the parallax luminosity and the asteroseismological luminosity is encouraging. Once the *HIPPARCOS* parallaxes are released it will be possible to check the asteroseismological luminosity estimates of other roAp stars.

¶ The other entry in the 'ν_3' column also emerges from an inconclusive analysis (that of Weiss' old 1983 data) and is of no help in discriminating between these two possibilities.

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