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The President's Social Security Plan: How Much Would You Pay to Participate?

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1 Introduction

The first modern social security program in the world was started in Germany in 1889. In 1935 the United States became the 55th country to initiate a similar program. Since then the social security program has become perhaps the most popular government program in the United States. Most developed countries now have some version of a social security program. Across the globe countries are grappling with demographic changes that threaten the viability of these programs.

The most common form of these programs is a pay-as-you-go system. Taxes are collected from current workers and that money is used to pay current retirees. No real assets are actually set aside to pay retirees in the future. The system worked well for many decades but problems are looming on the horizon. The primary problem in the United States is that the number of workers per retiree has dropped from about 16 in 1950 to about 3 currently and is projected to drop to about 2 by 2030. There are three main contributing factors to this problem. First, because of medical breakthroughs people tend to live longer. A person who reached the age of 65 was expected to live an additional 12.6 years in 1935. Today a person who reaches 65 is expected to live 17 more years and by 2040 that person is expected to live 19 more years. Second, many more people are deciding to retire early. The percentage of people in their early sixties who remain in the work force has dropped from 77 percent in 1960 to around 55 percent currently. The people who retire early begin to collect from the system (at admittedly lower rates) rather than contribute to the system. Third, birth rates have been falling since the 1950's which cause the work force to be replenished at a decreasing rate.¹

Currently the United States social security system collects more taxes than it pays out. Beginning in 2017 these surpluses are projected to become deficits. By 2030 the annual deficits are projected to be \$275 billion and \$719 billion by 2070. By 2077 deficits will have accumulated to a liability of an estimated \$25 trillion. Two obvious ways to avert these large deficits are to collect more taxes or pay lower benefits. The effect of either of these alternatives would be to lower the return that participants realize in the program. One estimate has a couple with two children born in 1932 receiving about a 4.74 percent return, after inflation, on their participation in pay-as-you-go system. The same couple born in 1976 will earn about 2.6 percent. Those born later will earn progressively less.²

In an attempt to remedy these problems many countries have changed their pay-as-you go systems to systems where some of the taxes collected are placed in an investment account in the

¹ One good source of background on social security is *Social Security Basics* (2002).

² Ibid.

taxpayer's name. These private accounts are typically invested in a mixture of bond and stock mutual funds. These accounts will partially fund the worker's retirement and will possibly lessen the government deficits in the future and provide a higher return for the participants. So far more than twenty countries have added these types of accounts to their social security systems. In the United States, the President has proposed allowing workers to divert 4 percent of their income to private accounts.

Are these private accounts a good idea for workers? There is a great deal of discussion and disagreement about the answer to this question. Of course, the answer partially depends upon what happens in the future. This paper attempts to show some conditions under which private accounts are a good idea or a bad idea for workers. We use an economic model to examine a representative worker and see how he might fare under differing circumstances. We use the idea of a "cash equivalent" to determine how much this individual would be willing to pay to be allowed to participate in the new private account system or how much he would need to be paid to agree to participate.

2 A simple model of social security³

In this model, we examine a representative individual whose work life is $[0, T]$, and retirement is $[T, \bar{T}]$. The retirement date is exogenous and the lifespan is certain.⁴ The economy-wide real, pre-tax wage rate is $w(t)$, which grows at rate x . The social security tax rate is θ , and is autonomous for simplicity. We assume that a fraction of the tax, α , flows into a private account, $k(t)$. Thus, we define α as the "rate of private funding" and, therefore, $\alpha\theta w(t)$ is the individual's annual contribution to the private account, and $(1-\alpha)\theta w(t)$ is the individual's annual contribution to the pay-as-you-go system. In the current, unfunded program, $\theta = 10.6\%$ and $\alpha = 0$, and under the President's current proposal, the tax rate stays the same (i.e., $\theta = 10.6\%$) but 4 percent of wage income would be used to fund private accounts, thus $\alpha = 4\% / 10.6\% = 38\%$, and the remaining 6.6 percent stays in the unfunded, pay-as-you-go system.

Thus, in a partially funded system the agent pays $\theta w(t)$ for all $t \in [0, T]$ and is paid an annual social security benefit $b(t) = (1-\alpha)\theta w(t)R$ for all $t \in [T, \bar{T}]$, where R is the ratio of the number of workers to retirees in the economy at the time benefits are received.⁵ And he is also able

³ Our model bears significant resemblance to Feldstein (1985).

⁴ See Feldstein (1974) for a discussion of the effect of social security on the date of retirement, and see Büttler (2001) for uncertain lifetime in a life-cycle model.

⁵ This formulation of the social security program makes a number of simplifying assumptions that are not central to the point of the paper. First, the supply of labor is perfectly inelastic so that the entire burden of social security taxation falls on the individual worker. Second, we exclude the possibility of budget surpluses or shortages. Third, we ignore the administrative costs of the social security program, which do not represent

to withdraw funds from his private account during retirement. At retirement, the balance in the private retirement account, $k(T)$, is

$$(1) \quad k(T) = \int_0^T \alpha \theta w(t) \exp(r(T-t)) dt$$

where r is the rate of return in the private account. We assume the individual is able to annuitize $k(T)$ so that the account is zeroed-out at the date of death, i.e., $k(\bar{T}) = 0$. This annuity, A , is the particular solution to the differential equation

$$(2) \quad \dot{k}(t) = rk(t) - A$$

with the boundary conditions given by (1) and $k(\bar{T}) = 0$. It can be shown that A is

$$(3) \quad A = \frac{k(T) \exp(r(\bar{T}-T))}{\int_T^{\bar{T}} \exp(r(\bar{T}-t)) dt}$$

2.1 Calibration

We consider an individual who begins working at age 25, retires at age 65 and dies at age 80. The variables T and \bar{T} are measured as the number of years after the individual begins working. Our benchmark calibration is:

- Market rate of return: $r = 3\%$
- Social security tax rate: $\theta = 10.6\%$
- Date of retirement: $T = 40$
- Date of death: $\bar{T} = 55$
- Initial wage rate: $w(0) = \$40,000$
- Growth rate of real wages: $x = 1\%$

(Nordhaus 1999) reports that the after-tax, real rate of return on US corporate capital is roughly 6 percent. On the other hand, the after-tax, real rate of return on Treasury Bills can sometimes approach zero percent. We split the difference and use 3 percent for our benchmark estimate, which is very close to the rate typically used in the life-cycle consumption literature (Gourinchas and Parker 2002, Bullard and Feigenbaum 2004, Feigenbaum 2005). The retirement part of the social security program in the US is funded by payroll taxes equal to 10.6 percent of earnings (up to a cap of \$87,900 in 2004). The liability is divided evenly between workers and employers, but if the supply of labor is perfectly inelastic the worker pays the full 10.6 percent;

a large portion of the total social security budget. Fourth, the taxation of benefits is ignored since in the real world these revenues are channeled right back into the social security program. Fifth, we focus only on social security *retirement* benefits and ignore social security *disability* benefits.

thus, we assume $\theta = 10.6\%$. And our benchmark estimate for the rate of real wage growth, 1 percent, is the average annual growth in real earnings in the US over the 10-year period ending in December 2004.⁶

2.2 IRR

The internal rate of return (IRR) on the private account is equal to the market rate of return, r . If the market rate of return exceeds the internal rate of return in an unfunded program, switching from an unfunded to a partially funded system should provide greater retirement wealth for the individual, for a given level of tax collections.

Thus, it is instructive to calculate the internal rate of return in an unfunded program. To do so, we set α , the rate of private funding, to zero and we find the interest rate, i , that solves the following equation

$$(4) \quad \int_0^T \exp(i(T-t))\theta w(t)dt = \int_T^{\bar{T}} \exp(i(T-t))\theta w(t)Rdt$$

The left side of (4) gives the value of all social security tax collections (valued at the date of retirement) and the right side is the amount needed at retirement to sustain withdrawals equal to $b(t) = \theta w(t)R$, the social security benefit. The internal rate of return in an unfunded program is the value of i that solves (4). For our baseline calibration with $R = 3$, we obtain $i = 1.42\%$. Over the next few decades, the ratio of workers to retirees is scheduled to fall to 2, due in part to declining birth rates and increasing life-expectancies. In this case, $i = -0.1\%$.

It is important to note that the internal rate of return does not depend on the size of the program, i.e., the social security tax rate θ can be canceled from both sides of equation (4), leaving the internal rate of return as an implicit function of other parameters such as the wage growth rate and the ratio of workers to retirees.

Figures 1–3 show the break-even rates of real wage growth under alternative assumptions about the market rate of return and the ratio of workers to retirees.⁷ In Figure 1, we assume that market rate is 3 percent. When the ratio of workers to retirees is 3, and the economy-wide rate of real wage growth is greater than 2.6 percent, the internal rate of return in the pay-as-you-go program will be higher than the market rate of return. When the IRR lines are above the 3 percent

⁶ The Bureau of Labor Statistics, Series Id: CES0500000049.

⁷ There are a few technical points regarding all three figures that should be emphasized. First, the IRR lines are 45-degree lines, indicating that a 1 percent increase in the rate of real wage growth leads to a 1 percent increase in the IRR. Second, the IRR is greater than the rate of real wage growth under current demographic conditions ($R = 3$), and is less under future demographic conditions ($R = 2$). This is seen by noting where the IRR lines touch the vertical-axis. Third, the ratio of workers to retirees, R , scales the IRR line up or down, but does not affect its slope.

horizontal line the current pay-as-you-go program dominates the new proposed program. If the ratio falls from 3 to 2, the real wages must grow at 4.1 percent or more for the pay-as-you-go program to have a higher internal rate of return.

Figure 1. Break-Even Rates of Real Wage Growth
(Market rate of return, $r = 3\%$)

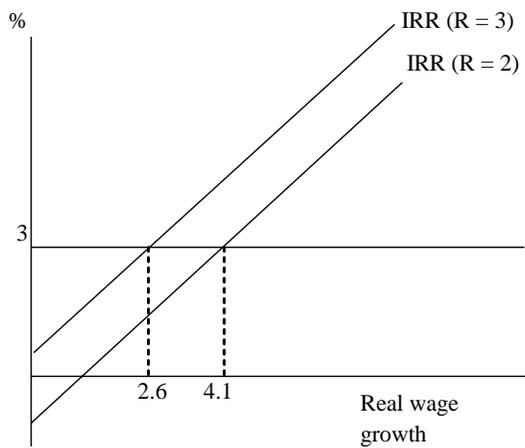


Figure 2. Break-Even Rates of Real Wage Growth
(Market rate of return, $r = 6\%$)

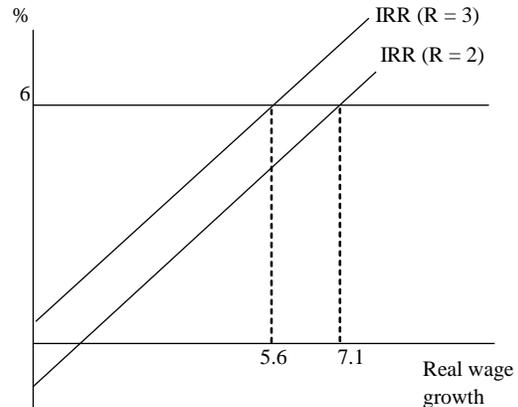
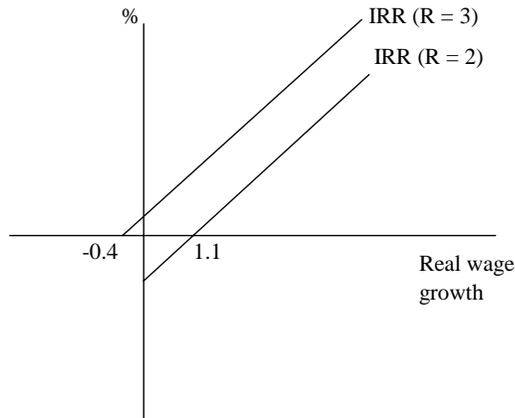


Figure 3. Break-Even Rates of Real Wage Growth
(Market rate of return, $r = 0\%$)



In Figures 1-3, the 45-degree lines show the IRR of the pay-as-you-go program for varying levels of real wage growth. Each figure has two IRR lines, one for current demographic conditions ($R = 3$) and one for future demographic conditions ($R = 2$). In Figure 1, the market rate of return is set at 3.0 percent, our benchmark. When the IRR is above the 3.0 percent line this indicates levels of wage growth for which the pay-as-you-go program is better than the private account program. Figures 2 and 3 are similar except that the market rate of return is set to 6.0 percent and 0.0 percent respectively. The graphs show that as the market rate of return increases, higher levels of wage growth are needed for the pay-as-you-go program to break even.

Figure 2 shows similar results under the assumption that the market rate of return is 6 percent. Here we see that the rate of real wage growth must be extremely high, 5.6 percent if $R = 3$ and 7.1 percent if $R = 2$, in order to render an above-market internal rate of return. However, in the case where the market rate of return is zero, Figure 3 shows that the pay-as-you-go program appears more favorable. In this case, when $R = 3$, wage growth only needs to be above -0.4 percent for the pay-as-you-go program to have a higher IRR. When $R = 2$ a wage growth over 1.1 percent will provide the pay-as-you-go program a higher return.

Of course, the future rate of real wage growth is uncertain, but we can look to the US experience over the last couple of decades as a point of reference. Table 1 provides summary statistics on the wage growth rate, taken from The Bureau of Labor Statistics. For the 30-year period ending in December 2004, the highest annual rate of real wage growth was 2.6 percent. Thus, even if we expect wages to grow in the future at a rate equal to the 30-year high, Figures 1 (i.e., where the market rate of return is 3 percent) suggests that a pay-as-you-go program will just barely break even under current demographic conditions ($R = 3$) and will have a below-market rate of return under future conditions ($R = 2$). If the market rate of return is 6 percent (e.g., as in Figure 2), a pay-as-you-go system will earn a below-market rate of return unless real wages in the future consistently grow at significantly faster rate than the 30-year high of 2.6 percent.

Table 1. Annual Rates of Real Wage Growth in the US*

10-year high	2.6%
10-year low	0.0%
10-year average	1.1%
20-year high	2.6%
20-year low	-2.2%
20-year average	0.2%
30-year high	2.6%
30-year low	-4.9%
30-year average	-0.1%

These data are from the BLS, Series Id: CES0500000049

However, if the market rate of return is lower, say zero percent, one can more easily justify a pay-as-you-go program. In this case, the internal rate of return is above zero as long as the rate of real wage growth is higher than -0.4 percent and 1.1 percent under current and future

demographic conditions, respectively (see Figure 3). From table 1, we see that the 10, 20, and 30-year averages all exceed -0.4 percent, implying that under current demographic conditions ($R = 3$), the internal rate of return in a pay-as-you-go program will be greater than the market rate (when the market rate is zero, as in Figure 3). However, under future demographic conditions ($R = 2$), even if the market rate is zero, it seems questionable whether a pay-as-you-go system can outperform the private account program. This is true since the 10, 20 and 30-year average growth rates for real wages are *no greater* than the break-even rate of 1.1 percent shown in Figure 3.

2.3 Cash equivalents

In this section we estimate the difference in the net present value of an unfunded, pay-as-you-go, program and a partially funded program, and we call the difference a “cash equivalent” or a willingness to pay to switch from an unfunded program to a partially funded program.⁸

The net present value of an unfunded program, NPV_{uf} , is equal to

$$(5) \quad NPV_{uf} = \int_T^{\bar{T}} \exp(-rt)\theta w(t)Rdt - \int_0^T \exp(-rt)\theta w(t)dt$$

and the net present value of a partially funded program, NPV_{pf} , is equal to

$$(6) \quad NPV_{pf} = \int_T^{\bar{T}} \exp(-rt)(1-\alpha)\theta w(t)Rdt - \int_0^T \exp(-rt)(1-\alpha)\theta w(t)dt \\ + A \int_T^{\bar{T}} \exp(-rt)dt - \int_0^T \exp(-rt)\alpha\theta w(t)dt$$

In (6), the first two terms on the right give the net present value of the unfunded component. The last two terms give the net present value of the funded component, which is zero (this is intuitive, but it can be demonstrated by substituting (1) and (3) into (6)). Thus, we can rewrite (6) as

$$(6') \quad NPV_{pf} = \int_T^{\bar{T}} \exp(-rt)(1-\alpha)\theta w(t)Rdt - \int_0^T \exp(-rt)(1-\alpha)\theta w(t)dt \\ = (1-\alpha)NPV_{uf}$$

Thus, using (5) and (6') we can write the willingness to pay to switch from an unfunded to a partially funded program as ⁹

⁸ The cash equivalents that we estimate correspond to an individual that is just entering the workforce.

⁹ Although we have not explicitly mentioned utility or any type of optimization process, our estimates of cash equivalents are equal to compensating variations derived from the Modigliani-Freidman, Permanent-Income Life-Cycle Model (PI-LC Model). In the basic PI-LC Model, the individual calculates the present value of all future income (after-tax wage income plus social security income) and then spreads this wealth across the life-cycle in a manner that maximizes utility. Although it may be obvious to experts in the life-cycle

$$(7) \quad \begin{aligned} NPV_{pf} - NPV_{uf} &= (1 - \alpha)NPV_{uf} - NPV_{uf} \\ &= -\alpha NPV_{uf} \end{aligned}$$

3 Results for the cash equivalents

Table 2 shows cash equivalents for different levels of the parameters. In each row the cash equivalents are presented for the current ratio of workers to retirees ($R = 3$) and for the projected future ratio ($R = 2$). The cash equivalents for the benchmark parameters are shown in the center column. When $R = 3$ a worker would be willing to pay \$16104 and when $R = 2$ that amount increases to \$25,420. Cash equivalents for lower parameter values are given to left and higher values to the right. The first thing to notice in the table is that almost all of the cash equivalents are positive. This indicates that under most circumstances presented, a worker would be willing to pay to be allowed to participate in a program with private accounts. Table 3 gives the levels of variables that are needed for the pay-as-you-go program to be equal to the private account program. These break-even levels are given for our current ratio of workers to retirees ($R = 3$) and for the projected future ratio ($R = 2$). The implications of the change in each variable are discussed below.

Table 2. Cash Equivalents -- What a worker would pay to switch from the current pay-as-you-go system to the proposed system of partial private accounts

	Bench- mark						
Growth Rate of Wages	0	0.3%	0.7%	1.0%	1.3%	1.7%	2.0%
R = 3	19,807	19,030	17,575	16,104	14,239	11,015	7,931
R = 2	26,628	25,731	25,664	25,420	24,971	23,978	22,870
Market Rate of Interest	0%	1%	2%	3%	4%	5%	6%
R = 3	-37,194	-8,000	7,931	16,104	19,807	20,993	20,814
R = 2	1,435	16,000	22,870	25,420	25,628	24,637	23,099
Rate of Private Funding	0%	10%	20%	38%	50%	75%	100%
R = 3	0	4,268	8,535	16,104	21,338	32,006	42,675
R = 2	0	6,736	13,473	25,420	33,682	50,523	67,364
Age of Death	65	70	75	80	85	90	95
R = 3	44,054	33,791	24,506	16,104	8,501	1,622	-4,602
R = 2	44,054	37,212	31,022	25,420	20,352	15,766	11,607

consumption area, we demonstrate in the appendix that the compensating variation that renders the PI-LC individual indifferent between participating and not participating in the President's plan is exactly equal to the difference in the net present value of an unfunded and a partially funded social security program.

Table 3. Break-Even Parameters -- Where workers are indifferent between the current pay-as-you-go system and the proposed system of partial private accounts

	Break-Even Value	Change from Benchmark
Growth Rate of Wages		
R = 3	2.6%	1.6%
R = 2	4.1%	3.1%
Market Rate of Interest		
R = 3	1.4%	-1.6%
R = 2	-0.1%	-3.1%
Age of Death		
R = 3	91	11
R = 2	112	32

3.1 The rate of wage growth

In the first column of Table 2 varying levels of wage growth are presented. Under all levels of wage growth shown, workers would be willing to pay to participate in the private account program. That is, they would have larger present value of wealth from the private program. As the rate of wage growth increases, however, workers become less eager to participate in the private account program. It is also worth noting that as the ratio of workers to retirees drops from 3 to 2 workers become more willing to opt for private accounts. Table 3 shows the same result presented in Figure 1. When $R = 3$ a wage growth rate above 2.6 percent is required for individuals to be indifferent to the pay-as-you-go program. When $R = 2$ the break-even rate of wage growth increases to 4.1 percent.

3.2 The market rate of return

At the benchmark market rate of return, 3.0 percent, workers would be willing to pay to participate in the private account program. As the market rate of return increases people generally become more eager to participate in the private account program.¹⁰ This is intuitive because the more people can earn on their private accounts the happier they are to participate in them. Conversely, at very low market rates the private accounts lose their luster. If the market rate of return is at 1.0 percent or 0.0 percent, and $R = 3$, workers would be willing to pay **not** to

¹⁰ It is interesting to note that the cash equivalent does *not* increase monotonically with increases in the interest rate. For example, when $R = 3$, and the interest rate is zero, the cash equivalent is negative because the non-discounted benefits from a pay-as-you-go program exceed the costs. However, as the interest rate rises, the present value of the benefits will decrease in absolute size quicker than the present value of the costs (because benefits are further in the future), and the cash equivalent becomes positive. Next, the gap between discounted benefits and costs reaches a maximum between 5 and 6 percent (when $R = 3$). As the interest rate continues to grow large the discounted value of the benefits and costs both approach zero, and the cash equivalent therefore declines toward zero in the limit (i.e., as r approaches infinity).

participate in the private account system and thus the cash equivalents are negative. Even though the IRR of the pay-as-you-go system is quite low, as reported above, at very low market rates of return the current system can look attractive. Table 3 shows that the break-even market rate of return is 1.4 percent under current demographic conditions. When the ratio of workers to retirees drops to 2, however, the break-even market rate of return becomes -0.1 percent. At this level of R , any positive market return will make workers prefer the private account system.

3.3 Age of death

In the pay-as-you-go program workers who die young get a very low IRR. The worst case scenario is someone who pays into social security all of their working life and then dies at age 65. They will not receive any benefits and their effective IRR is negative infinity. Table 2 shows that as the age of death increases the amount that people are willing to pay to participate in private accounts diminishes. In fact a person who lives to age 95, if $R = 3$, would not want to participate in private accounts. The break-even age is at 91. This is because as you live longer you receive more and more benefits and the IRR of the pay-as-you-go program increases. When R drops to 2, Table 2 shows that the break-even age of death increases dramatically to 112. Virtually every one would prefer private accounts in this case.

One issue discussed about the current pay-as-you-go program is how different demographic groups fair. Actuaries have shown that minority groups and those with less education tend to die earlier. Our results, from Tables 2 and 3, show that those who die early are the ones most willing to pay to opt into private accounts. These groups fair the worst under our current system and would benefit most from a switch to private accounts.

3.4 Passing social security benefits on to your heirs.

In the current pay-as-you-go system you are not allowed to bequeath to your heirs the payments you have made over your working life. In our current system if one spouse dies the other is entitled to the larger of their own benefits, or the benefits of the late spouse. Thus if you are eligible for social security benefits and you spouse dies, all or most of the late spouses benefits are lost. In the private account system, the amount remaining in your account when you die may be willed to your heirs. This is related to the "age of death" section above. Those who die the youngest would benefit most from being able to will their accounts to their heirs and those who live the longest would benefit the least from this provision.

3.5 The rate of private funding

In the third row of Table 2, cash equivalents for different levels of private funding are shown. If the rate of private funding is 0 percent then we have our current system and a cash equivalent is irrelevant. The benchmark is set at 38 percent which represents the President's 4 percent private account contribution divided by the current 10.6 percent. A 100 percent rate of private funding would indicate a totally private system with no pay-as-you-go system remaining. Notice that as the percentage of private funding increases, workers become more willing to pay to participate. The intuition here is obvious. If workers would pay a little to participate in a system that uses partial private accounts, they should be willing to pay more to participate in a system that makes greater use of private accounts.

4 Conclusions

In this paper, a simple life-cycle framework was used to determine under what conditions a worker would benefit from switching from a pay-as-you-go social security system to one that includes some private accounts. Under most conditions a worker would be willing to pay to be allowed to participate in a private accounts system. The willingness to pay to participate increases as, the rate of wage growth decreases, as the market rate of return increases, as the age of death decreases and as the rate of private funding increases. Using the current ratio of workers to retirees in the US (3 workers per retiree), we calculate that if the real market rate of return is very low, less than 1.4 percent, the current pay-as-you-go system would be preferable. But as the ratio of workers to retirees declines from 3 to 2, the break-even level of the market rate declines to -0.1 percent. For workers who live a very long time, at least 91 years, the pay-as-you-go system is now preferable. Here again, however, as the ratio of workers to retirees declines from 3 to 2 that break-even age increases dramatically to 112 years of age. With the exception of the two situations just mentioned, the switch to a system that allows for some contributions to private accounts seems to be beneficial to most workers.

References

- Bullard, James and James Feigenbaum (2004), A Leisurely Reading of the Life-cycle Consumption Data. Working paper.
- Bütler, Monika (2001), Neoclassical Life-cycle Consumption: A Textbook Example. *Economic Theory* 17, 209-221.
- Dominitz, Jeff, Charles F. Manski, and Jordan Heinz (2003), "Will Social Security Be There For You?": How Americans Perceive Their Benefits. Working Paper.
- Feigenbaum, James (2005), Can Mortality Risk Explain the Consumption Hump? Working paper.
- Feldstein, Martin (1985), The Optimal Level of Social Security Benefits, *Quarterly Journal of Economics*, 100(2), 303-320.
- Feldstein, Martin (1974), Social Security, Induced Retirement and Aggregate Capital Accumulation. *Journal of Political Economy*, 82, 905-926.
- Gourinchas, Pierre-Olivier and Jonathan A. Parker (2002), Consumption Over the Life Cycle. *Econometrica* 70, 47-89.
- The Heritage Foundation (2002), *Social Security Basics*, WebMemo #143, [http://www.heritage.org/Research/Social Security/wm143.cfm](http://www.heritage.org/Research/Social%20Security/wm143.cfm).
- Nordhaus, William D (1999), Discounting and Public Policies That Affect the Distant Future, in *Discounting and Intergenerational Equity*, (Paul R. Portney and John P. Weyant, eds.), Resources for the Future. Washington, DC.

Appendix

The purpose of this appendix is to show that the cash equivalents calculated above are equal to compensating variations from the basic Permanent-Income Life-Cycle Model. We rely on notation already introduced in the body of the text, and the basic model is as follows.

Inflation adjusted (real) consumption spending for a representative individual at time t is denoted by $c(t)$, and the agent receives utility at time t according to $U(c(t)) = \ln c(t)$. The rate of impatience or time preference is ρ . We start by analyzing the optimal consumption path for an individual who participates in an unfunded program. At time zero the individual enters the workforce and solves to following problem¹¹

$$(A1) \quad \max : \int_0^{\bar{T}} \exp(-\rho t) \ln c(t) dt$$

subject to

$$(A2) \quad \dot{k}(t) = rk(t) - c(t)$$

$$(A3) \quad k(0) = \int_0^T \exp(-rt) w(t) dt + NPV_{uf}$$

$$(A4) \quad k(\bar{T}) = 0$$

where $k(t)$ is the individual's savings account. Equation (A2) states that any income that is not consumed flows into the asset account. In (A3) we take advantage of the well-known principle of life-cycle optimal control models that cash-flow *timing* is relevant only to the extent it affects the present value of lifetime wealth. The first term on the right on (A3) is the present value of pre-tax wage income, and the second term is the net present value of an unfunded social security program.¹² Thus, we compress wages and social security income into an initial capital endowment. The solution

¹¹ This simple specification of preferences leads to time-consistent behavior – the individual will stick to the planned consumption profile.

¹² This writing of the wealth constraint implies that the individual perfectly anticipates his social security benefits. In an unfunded system, benefits are a function of the wage rate, the ratio of workers to retirees, and the social security tax rate. Each of these items, especially the last, may be difficult to predict. Of course, since 1999 the Social Security Administration has mailed annual statements with projected benefits to all households above the age of 25, but these statements are based on current tax law and therefore may not be accurate. We recognize that the assumption of perfect foresight may be particularly questionable in this instance. Dominitz et al. (2003) document widespread pessimism about social security benefits among young workers, implying that young individuals may underestimate the level of benefits they will receive.

consumption profile is the same whether the individual is given an initial endowment equal to the present value of lifetime income, or if these payments are spread out over time.¹³

Simple application of the Maximum Principle gives the optimal consumption path as a function of initial wealth, $k(0)$:

$$(A5) \quad c(t) = \frac{\rho k(0) \exp((r - \rho)t)}{1 - \exp(-\rho T)}$$

In a partially funded system, instead of (A3) we have

$$(A6) \quad k(0) = \int_0^T \exp(-rt)w(t)dt + NPV_{pf}$$

The compensating variation, cv , that would render the individual indifferent between an unfunded and a partially funded program is the initial payment that equalizes the consumption paths across the two alternatives, thus, the cv is the solution to

$$(A7) \quad \int_0^T \exp(-rt)w(t)dt + NPV_{pf} - cv = \int_0^T \exp(-rt)w(t)dt + NPV_{uf}$$

which implies $cv = NPV_{pf} - NPV_{uf}$, which is identical to the cash equivalent estimates in Section 2.3.¹⁴

¹³ If payments are spread out over time, the problem becomes a two-stage optimal control problem with a switch in the state equation at retirement (a switch from wage to social security income). The solution to the two-stage problem is identical to the solution to our problem.

¹⁴ It is worth mentioning that the compensating variation from this control model is not a function of the rate of time preference. That is, the degree of impatience does not affect the individual's willingness to pay to switch plans. This is seen by noting that the rate of time preference is not found in equation (A7). In a well-known article, Feldstein (1985) establishes a link between impatience and individual welfare from participating in a pay-as-you-go program, and we could very easily establish the same link in our model if social security benefits were incorrectly estimated as in Feldstein's model.