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CENTER CONFIGURATIONS OF HAMILTONIAN CUBIC SYSTEMS

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ABSTRACT. Second order eyes of Hamiltonian cubic systems are classified into seven classes based on the orientation of the cycles within these eyes. They are further categorized into nine Conti classes based on the structure of the separatrix cycles that bound them. Examples of systems of each type are presented, and examples of third and fourth order eyes are also given. The classification is connected to part of Hilbert's sixteenth problem which asks for the possible relative positions of cycles in an autonomous polynomial system in the plane.

1. Introduction. Hilbert's sixteenth problem poses two questions for systems of differential equations of the form

$$\begin{cases} dx/dt = P(x,y) \\ dy/dt = Q(x,y), \end{cases} \tag{1.1}$$

where P and Q are polynomials of degree at most n in x and y. The first is to determine the maximum number of limit cycles, $H(n)$, that such a system may have. The second is to determine all the possible relative positions of limit cycles for each given degree n. Here we consider a related problem for the case $n=3$, but rather than limit cycles we consider the possible relative positions of the continuous bands of cycles that occur in Hamiltonian systems.

This is connected to the *weak Hilbert problem* or *Arnol'd problem* [**1**] which asks for a bound, $A(n)$, on the number of limit cycles that can be produced by perturbation having degree less than or equal to n from a Hamiltonian system of the same degree n. It is natural to also ask for the possible relative positions of limit cycles produced in this way. For quadratic systems ($n=2$), the Arnol'd problem has been answered by Horozov and Iliev [**8**] in the case of generic Hamiltonian systems which have a single center, and three saddles whose separatrices do not

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