Remote Heart Monitoring: A Predictive Modeling Approach for Biomedical Signal Processing

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(ABSTRACT)

Smart healthcare is an emerging field with the goal of harnessing technological advances to enhance healthcare quality. Several research projects in recent years are devoted to design of electronic devices and networking platforms to facilitate technology-based health service. One important paradigm in smart healthcare is developing tools for biomedical signal processing. Biomedical signals can directly reflect the information about patient health and therefore have been widely investigated by the research community. The essence of most signal analysis systems is to process a large training dataset and build a reference model to asses the health status of new patients. While the majority of these methods focus on improving classification performance on a collection of signals in large datasets, the predictive modeling of biomedical signals is rarely emphasized. In this work, we go one step beyond the conventional methods and intend to predict potential upcoming abnormalities before their occurrence. The approach is to build a patient-specific model and identify minor deviations.

To enable an accurate deviation analysis, a controlled nonlinear transformation is proposed to reshape the feature space into a more symmetric geometry. We applied the developed algorithms on Electrocardiogram (ECG) signals and the results confirm the effectiveness of the proposed method in predicting upcoming heart abnormalities before their occurrence. For instance, the probability of observing a specific abnormality class increases by 10% after triggering a yellow alarm of the same type. This approach is general and has the potential to be applied to a wide range of physiological signals.

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Chapter 1

Introduction

1.1 Background and Motivation

Heart-related mortality rate has been recently increasing dramatically due to the aging of population, chronic cardiovascular diseases and increasing life stress and pace of modern life [1]. According [2], cardiac diseases are the most common cause of sudden cardiac death (SCD) with 250 000 to 300 000 mortalities in the U.S. every year accounting for 14.7% of total deaths [2]. As World Health Organization reported, 31% of global deaths are related to cardiovascular diseases (CVDs) [3]. These facts fully reflect that heart diseases are threatening the general health of human beings. Since death from CVD can occur in most cases without prior warning and obvious symptoms, enabling a timely treatment of heart diseases is very beneficial. For this purpose, prevention principles and guidelines which cover age, family history and other potential risk factors causing CVD are deployed in most clinical modeling methods [4]. However, these methods require complex manual analysis by trained physicians. Taking this issue into consideration, developing a cost-effective automatic analysis for CVD prevention based on computer is a critical need. More specifically, since most CVDs are accompanied with arrhythmia, accurate and timely recolonization of arrhythmia is a key factor for effective prevention of heart disease.

Electrocardiogram (ECG) is the most common way of monitoring hearth functionality, which contains abundant physiological and pathological information that reflects the heart rhythm and status of various parts of the heart. ECG signals are recorded for the first time by Waller in 1887 [5]. He recorded signals generated by electrical activities of heart as a time series. As a noninvasive examination method, ECG signals are known to be highly reliable in reflecting functionality of heart. For this reason, ECG signal processing has become one of the most conventional technologies for clinical examination, radiation and ultrasonic inspection in modern hospitals and clinics, serving as an important reference for doctors' diagnosis of heart disease [6].

The traditional diagnosis based on ECG signal is mainly performed by physicians through visual observation and interpretation. However, the approach is costly and impractical when a continuous monitoring of patients is required (e.g. to recognize CVD conditions). There are tremendous ECG records generated everyday, all demand for timely diagnosis and analysis. Due to the limitations in the access to experienced physicians, automated ECG classification systems have been introduced and became popular soon afterwards to generate real-time analysis result and provide additional information to physicians.

Several computer-based automated classification algorithms have been developed by researchers in the last decades to minimize human intervention or to assist physicians with more accurate diagnosis by reducing human mistakes [7–17]. Moreover, with the emerging applications of smart health and smart cities, a constant monitoring and analysis of ECG and other physiological signals with direct experts' intervention deems impossible. Therefore, applying conventional classification algorithms on biomedical signals remains challenging, especially for applications of spontaneous disease detection. Two important properties of cardiovascular disease include the wide range of its causing factors and the difficulty of recognizing some implicit symptoms before occurrence [18, 19]. Failing to predict life threatening CVDs is the principal cause of high mortality for patients with heart disease. A timely prediction of hearth abnormalities before their actual occurrences would enable a therapeutic intervention before the condition becomes detrimental, hence it minimizes the risk of mortality. Nevertheless, the majority of developed conventional ECG classification systems are only able to detect abnormalities when they occur. To the best of our knowledge, no research work is devoted to the prediction of heart abnormalities ahead of time, which is the main focus of this project [20, 21].

Another important property of ECG waveforms is their inherent variability among different individuals under different physical and environmental conditions including but not limited to gender, age, body-mass index, elevation and air pressure, humidity, temperature, and etc [22, 23]. Conventional classification algorithms do not easily generalize, when applying to different patients' records [14]. Due to the inter-patient variation in ECG signals and the complexity of cardiac pathological information analysis, most of the existing ECG analysis software only serve as auxiliary tools for physicians. The final results of diagnosis still depend on manual labeling by cardiologists. Recently, several novel patient-specific ECG analysis methods are proposed. Broadly speaking, in these methods, systems parameters are adaptable to the individual ECG signal properties [12–17]. Some algorithms combine cardiologists' manual annotations with automatically generated labels and train personal classifier with updated labels for each individual [12–14]. This design still requires experts' assistance in order to accurately classify ECG signals. Another commonly used approach is to train a patient-specific classifier using only the patient's ECG signal. However, this method fails when a certain type of abnormal signal is not included in the limited personal ECG signal data.

The automatic analysis of ECG signals includes a wide range of techniques. In this work, we focus on overcoming the two drawbacks of existing automatic ECG classification systems, namely the failure in capturing patient-specific variability and the lack of predictive power. This research aims at improving the inter-patient classification performance and prediction capability of ECG-based diagnosis methods. Our proposed method can revolutionize the current practice of healthcare service by enabling early detection of heart abnormalities, with applications in the remote heart monitoring of high-risk people, senior people, and athletes. It also can significantly reduce the mortality rate of SCD by enabling time detection of symptom-free heart abnormalities.

1.2 ECG and Arrhythmia

Electrocardiogram is widely used to monitor the electrical activities of heart and assist diagnosing fatal cardiac diseases. In order to design algorithms specifically for ECG analysis, it is important to develop an insightful perception of the functionality of heart and ECG waveforms.

1.2.1 Characteristics of ECG signal

An ECG signal reflects the properties of periodical electric signals generated by a heart. Fig. 1.1 demonstrates the typical signal waveform for a cardiac cycle (i.e. a heartbeat), which is usually composed of three main waves including P wave, QRS complexes and T wave. These waves corresponds to different physiological activities of the heart. P waves are generated by atrial depolarization which represents the process of pumping blood to ventricles. QRS complexes as the most significant electric activities are caused by the *ventricular* contraction,



Figure 1.1: A typical cardiac cycle in ECG signal with five characteristic waves.

which is the process of pumping blood to lungs and the rest of the human body. Finally, T waves are the results of *ventricular* repolarization, which is a required recovery process before the following cardiac cycle. Accurate detection and segmentation of each wave is necessary for a profound ECG analysis. The waves are usually represented by their peak locations, also called fiducial peaks. By detecting the most significant peak within QRS complexes (i.e. R peak) automatic algorithms are able to discriminate between the two adjacent cardiac cycles. The interval between two R peaks is called RR interval, which is also the inverse of heart rate. Fig. 1.1 represents a typical cardiac cycle with the aforementioned intervals.

1.2.2 MIT-BIH Arrhythmia Database

Arrhythmia is related to various morbid behaviors of heart. Generally speaking, arrhythmias consist of two main categories: supraventricular and ventricular. Ventricular ectopic beats imply abnormal activities in the ventricles while *supraventricular* ectopic beats are related to the *atria* [24]. Both categories contain fatal abnormal beats, which may lead to death [25]. Therefore, in order to help researchers standardize the analysis and research works related to ECG classifiers, the Association for the Advancement of Medical Instrumentation (AAMI) has proposed a set of recommendations for reporting ECG classifier performance [26]. According to these recommendations, MIT-BIH Arrhythmia Database (MITDB) is regarded as a standard database to train and test ECG classifiers in the last two decades. MITDB is a public database which is available on Physionet.com [27] since 1997 [28]. There are 48 records collected from 47 individuals in this database. Each record contains two channels of ECG raw signals along with annotations for each cardiac cycle. Annotated labels include 16 types, as shown in Table 1.1. Among these two channels, MLII is more informative and commonly used in automated ECG analysis systems. We also adopt the signal from this channel as system input [29]. Cardiac cycles are determined by the locations of R peaks. The sampling frequency of MITDB is 360Hz and the signal frequency spans from 0.1 to 100 Hz.

Following the recommendations by AAMI, the original annotations of MITDB, consisting of 16 different arrhythmia types, are further grouped into 5 major classes: class N(normal and bundle branch block beat types) class V(ventricular type), class S(supraventricular type) and class F(fusion of normal and Ventricular types). The class Q which includes unclassified and paced beats are discarded due to the limited number of samples. Table 1.1 summarizes the mapping from 16 original types that include cycles of this type. Therefore, only 4 remaining types (N, V, S, F) are typically used.

Standard Types by AAMI	Original Types in MITDB Annotation
Ν	NOR, LBBB, RBBB, AE, NE
V	PVC, VE, VF
S	APC, AP, BAP, NP
F	VFN
Q	PACE, FPN, UN

Table 1.1: Mapping from 16 original types in annotation to the standard 5 types recommended by AAMI

1.3 Problem Statement

ECG signals are investigated broadly by researchers to design automated non-invasive diagnosis methods and real-time monitoring systems [17, 30, 31]. As described in section 1.2, a majority of current methods suffer from two main challenges: i) failure to capture interpatient variability and ii) incapability of early detection and prediction.

In conventional classification systems, the training dataset is typically composed of records collected from different patients with experts' annotations per heartbeat. In order to unify the records from different patients, most of the conventional classification algorithm mix heartbeat samples from different individual ECG records and cluster the pooled ECG dataset simply based on the annotations of heartbeats. Since the classification performance is measured based on the comparison between the predicted labels with the annotate (true) labels for each sample, the classifiers are trained to improve the performance on pooled ECG data. While ECG signals shares similar morphologies, the signals from different patients demonstrate considerable variability as shown in Fig.1.2. Ignoring this difference will lead to inconsistent classification performance between patients. Therefore it's of significant importance to adjust classifier configuration according to patient-specific characteristics.

In addition to the inter-patient variability, majority of ECG classification algorithms fail to provide a predictive capability, which refers to the power of triggering corresponding alarms



Figure 1.2: ECG signals of normal heartbeat from 15 different records in MIT-BIH reflect the inter-patient variability of ECG signal.

before the occurrence of abnormalities. Typically alarms represent significant distortions in the ECG morphology which reflect a severe heart abnormality. The rest of heartbeats considered as *normal* beats. However, an abnormal beat may include mild distortions that can be indicative of a problem, while it is not severe enough to call a *red alarm*. In this work, we represent these minor deviations with *yellow alarm* and use them to predict real abnormalities as *red alarms*, before their actual occurrence.

Fig. 1.3 illustrates this concept, where observation of minor deviations from the patientspecific normal trends (*yellow alarms*) can be predictors of upcoming severe abnormalities (*red alarms*). Supporting results are provided in section 3.4 and section 4.6. Therefore, a method to quantify the level of signal similarity to abnormalities should be incorporated into the ECG classification system.



Figure 1.3: A *yellow alarm* represents a minor abnormality which can be indicator of an upcoming severe abnormality of the same type in terms of a *red alarm*.

1.4 Literature Review

Automatic analysis of ECG signals refers to the entire process spanning from the acquisition of signals to the classification of samples. This process can be divided into five stages: ECG signal acquisition, preprocessing, fiducial peak detection and segmentation, feature extraction and predictive modeling (Fig. 1.4). Different research works focused on one or multiple stages of the automatic analysis system. Since the main objective of this work is addressing problems in classification algorithms, the literature review in this section focuses on studying existing methods proposed for stages before classification, conventional classification algorithms along with patient-specific classification systems.



Figure 1.4: General structure of ECG analysis system.

1.4.1 ECG Signal Preprocessing

During data acquisition, the ECG signal may be affected by different kinds of noise including physiological noise (e.g. myoelectricity noise, breathe interference etc.) and non-physiological noise (e.g. power-frequency interference and electrode impedance interference) [32]. These noises often interfere with the informative signals and thus influence the ECG classification results. Therefore, ECG signal preprocessing mainly focuses on the suppression of noise and interference terms in the ECG signal.

The ECG signal is in millivolt (mV) level with a central frequency ranging from 0 to 40 Hz [33]. Due to the relatively low signal to noise ratio of ECG signals, signal preprocessing is a necessary step before classification. Therefore, various methods are proposed to eliminate noise and other artifacts from the ECG signal [32–39].

Generally speaking, ECG signal preprocessing methods include finite impulse response (FIR) filtering, adaptive filtering, and modern signal processing filter methods such as wavelet

transforms and neural networks [32, 35, 36, 39]. YW Bai *et al.* compared different notch filters and concluded that equiripple notch filter outperforms other methods in terms of noise reduction and CPU time [35]. Lian *et al.* [34] designed a multiplier-free finite impulse response (FIR) filter to surpress biological and environmental noises with a low power consumption. Sayadi *et al.* proposed a modified extended Kalman filter with estimated hidden state variables to perform denoising and compression at simultaneously [36]. Park *et al.* designed a wavelet-based adaptive filter to reduce S-T segment distortion due to the baseline drift and compared its performance with general adaptive filters [37]. A general conclusion is that the performance of wavelet adaptive filtering is usually higher than generic adaptive filters. In [38], the authors combined wavelet decomposition with Wiener filtering to filter out the noise by thresholding, which is proved to outperform other thresholding denoising methods. Regarding various wavelet basis functions, Singh *et al.* studied an optimal selection of basis functions for ECG signal denoising [32]. By comparing the classification root mean square error using the same classifier and different denoising methods, they concluded that Daubechies filter of order 8 is the best choice for ECG classification systems.

1.4.2 Fiducial Peak Detection and Segmentation

Fiducial Peak Detection and cardiac cycle segmentation are the basis of extracting important information from ECG signals, since a ECG record is usually a continuous time signal. This signal can be split into smaller intervals, each of which representing to one cardiac cycle. Each cardiac cycle can be viewed as an independent signal and is associated with a separate label to represent the heart function during the corresponding interval. The accuracy and reliability of this stage directly determine the final performance of the overall diagnosis and analysis. Fiducial peak detection, which is also called ECG signal delineation, aims at localizing five characteristic peaks within one cardiac cycle. The most significant peak is the QRS complex consisting of Q, R and S peaks. The other two fiducial peaks include P wave before the QRS complex and T wave after the QRS complex. As shown in Fig.1.5, these five characteristic waves along with the onset and offset of the QRS complex are often used to present a cardiac cycle.



Figure 1.5: Fiducial peaks within one cardiac cycle.

The QRS complex is the most prominent wave and it contains the majority of the information of a ECG signal; therefore, most of the ECG delineation methods detect QRS complex prior to the detection of other peaks. Afonso *et al.* proposed a method using filter banks to detect QRS complexes [40]. In this method, the signal is decomposed to several frequency bands. Fiducial peaks are thus detected using its morphological features in the decomposed signals. Sadhukhan *et al.* proposed a method of detecting R peak by thresholding the double difference signal of ECG data and comparing the relative amplitude within QRS region [41]. The performance of this method is validated using clinical ECG signals and has been proven to be promising. Some advanced machine (ML) learning techniques are also deployed to detect QRS complexes. In [42], *Support Vector Machine* (SVM) is used to train a predictive model for QRS complex detection and achieved a classification accuracy of 99.93%. Other ML methods such as *Hidden Markov Models* (HMM) are also investigated and proven to be effecient in modeling and detecting characteritic peaks in ECG signals [43]. Wavelet decomposition is also frequently adopted for signal delineation due to the morphological similarity between wavelet basis functions and QRS complexes. As the QRS complex power spectrum is centered at the range of 5 Hz to 30 Hz, the wavelet coefficients of the corresponding scale levels are frequently used for delineation purpose. In [44], QRS complexes are detected by thresholding wavelet coefficients at scales 1 to 4, then onset, offset and individual waves within QRS complexes are detected using the morphological characters of coefficient at scale 2. T and P waves are detected at scale 3 with a similar method approach. In the literature, some improvements have been proposed to eliminate false detection of R peaks by adding a fixed searching window of 160ms [45].

1.4.3 Feature Extraction and Classification

After localizing the fiducial peaks within a cardiac cycle, we proceed with the next step of extracting informative features of the signal, which collectively convey meaningful information about the signal properties. Since the objective of designing an automatic classification system is to precisely predict types of sample signals, feature selection is usually performed to obtain a better performance and reduce the computation cost [7-11].

As the most significant wave within an ECG signal, the information of QRS complexes are proved to be the most important features for ECG classification systems. Lagerholm *et al.* decompose QRS complexes with a set of Hermite basis functions and the decomposition coefficient are deplyed as ECG features to train a Self-Organizing Map (SOM), which achieved an average error rate of 1.5% for 16 ECG types [7]. Prasad *et al.* used discrete wavelet transform (DWT) to extract RR intervals between the current beat and the previous or next beats. The two RR-intervals serve as input for training a neural networks, which achieves the average accuracy of 96.77% in classifying 13 different arrhythmia types. De Chazal *et al.* proposed two set of features: morphology and heartbeat interval features. They used different combinations of these features combined with Linear Discriminant Analysis to classify ECG signals into five arrhythmia types and selected the optimal feature set according to classification accuracy [9]. The results show that the sensitivity of detecting two major arrhythmia types can be improved by feature selection. R. Ceylan *et al.* included RR interval as the only ECG feature to train a fuzzy clustering neural network that achieved an average detection rate of 98.35% [10]. Osowski *et al.* proposed two set of features including *Higher Order Statistics* (HOS) and Hermite characterization of QRS complex to classify ECG signals with Support Vector Machine. Their final average error rate is at 1.82% [11].

1.4.4 Patient-Specific ECG Classification

The main drawback of the majority of aforementioned methods mentioned in the last section is the lack of inter-patient model adjustment. In order to generalize the ECG classification systems to clinical applications, several methods which are more robust to inter-patient signal variation are proposed to address this issue [12–17].

Hu *et al.* proposed a patient-specific Mixture of Experts (MOE) classifier by incorporating personalized annotations provided cardiologists in the local classifier [12]. This method achieves patient-adaption capacity but requires further input from human experts. This MOE approach achieved an accuracy of 94.0% for distinguishing *Ventricular* beats from the other non *Ventricular* types. Following the design of MOE, de Chazal and B. Reilly proposed an improved patient-adapting classifier by reducing the requirement of manual annotations to as few as 10 beats for training adaptive local classifier [13]. And Llamedo et al. designed an automatic classification system, which uses experts' assistance, but does not fully depend on the experts and can work independently [14]. By implementing a special block-based neural networks (BbNNs), Jiang et al. achieved accuracies of 98.1% and 96.6% in distinguishing *Ventricular* ectopic beats and *supraventricular* ectopic beats from other types [15]. In [16], particle swarm optimization (PSO) is combined with a neural network to optimize the network structure using patient-specific training data. Based on 1-D convolutional neural networks (CNN), Kiranyaz et al. proposed a flexible algorithm, which adjusts its parameters using information extracted from individual signals [17]. The classifier demonstrates consistent performance over different ECG records achieving an accuracy between 98% and 99% for distinguishing *ventricular* types from *non-ventricular* types. (Accuracy = 98.9% Sensitivity = 95.9% Specificity = 99.4%). While this approach outperforms the aforementioned classification algorithms as it does not require expert further annotations, its performance reduces for some rare abnormal classes.

1.5 Contributions

A crucial drawback of these recently proposed patient-specific classification systems is their failure in predicting abnormalities in advance. These methods aim at improving classification performance by comparing generated labels with a ground truth for each beat, hence ignore the relationship between the generated labels and upcoming abnormalities. While in many common applications, this approach generates satisfying results, it does not meet the needs of SCD prediction.

One of the main objectives of this work is to address the problem of forecasting by proposing the concept of *yellow* and *red alarms* and proving the fact that *yellow alarms* can be indicators of upcoming *red alarms*. *Yellow alarms* are defined through a novel deviation analysis which assesses the tendency of deviant normal alarms to one of the *red alarm*. In order to realize such a deviation analysis, the symmetry of different abnormality classes in the feature space is desired. We propose a novel controlled nonlinear transformation that maps the original feature space into a new space that presents the desired symmetry. To elaborate more on the symmetry of abnormal classes in the feature space, we assume that there are one normal class and multiple abnormal classes for a signal while latent states exists for some of the normal samples that represent slight deviations towards abnormalities. By distinguishing latent states, the designed automated system is capable to generating a *yellow alarm* which indicates a high probability of the presence of some upcoming abnormalities (*red alarms*) of the same type. Therefore, the contribution of this work can be summarized as:

- propose a novel self-configuring patient-adaptive framework which incorporates a personal classifier into the predictive modeling;
- utilize a kernel-based method as a spatial transformation with parameters optimized using multiobjective particle swarm optimization (MOPSO) for the purpose of deviation quantification;
- design a controlled spatial transformation with deterministic mapping function to optimize cluster topology for predictive analysis;
- propose a deviation quantification method based on cosine similarities, which is capable of generating *red alarms* for upcoming abnormalities.

1.6 Organization of Thesis

In the following chapters, details of the proposed classification framework are presented after reviewing the introductory concepts and related works. Chapter 2 provides a general information about the utilized ECG dataset and the general framework used in proposed classification system. Chapter 3 describes the details of nonlinear transformation with kernel methods and presents the experimental results using a kernel transformation. With the concept of nonlinear transformation, chapter 4 introduces an optimized spatial transformation with a novel deterministic mapping function. The experimental results for the spatial transformation method are presented section 4.6. Finally, the experimental results for the proposed methods in chapter 3 and chapter 4 are studied and compared. More importantly, the predictive capacity of the proposed system is studied and analyzed in this chapter. Based on the experimental results, we introduce some potential directions to further improve the system in terms of classification and predicting accuracy.

Chapter 2

Patient-Adaptive ECG Classification Framework

2.1 Introduction

The automatic ECG signal analysis has been a popular research topic for decades. Several academic research projects have proven that the advanced design and implementation of automatic ECG analysis methods are beneficial for timely detection and therapeutic intervention of heart disease. However, there are still some major challenges to be resolved, and automatic ECG analysis should reach a level of maturity and reliability before getting ready for clinical use. One of the most typical challenges is the inherent inter-patient variation of ECG waveforms, which leads to the inconsistent performances of ECG classification systems when applied to different patients under different conditions. In this chapter, the details of the patient-adaptable ECG classification method, as the core of the proposed framework, are presented.

The goal of the automatic ECG analysis is to determine the arrhythmia types for each signal sample. A continuous ECG signals are firstly segmented into individual intervals, each of which represents a heartbeat cycle, to be processed by the further stages of the proposed algorithm. Section 2.2 and section 2.3 in this chapter focus on the data preparation stage, which includes four steps: signal preprocessing, delineation, segmentation, and feature extraction. Following the data preparation, section 2.4 elaborates on the details of the proposed two-stage hierarchical classifier. The proposed classification system includes a novel method for patient-adaptation by gradually capturing the normal range for each individual. More specifically, in section 2.4, a dynamic normal cluster shaping method to achieve the rationalization property is discussed. One feature of this method is that the cluster can track the trend of a patient's normal ECG waveforms and dynamically adapt to it. In many clinical applications, the physicians need to monitor the long-term heart activities in real time. This dynamic adaptation feature enables the system to address the issue of the intra-patient temporal variation as well.

2.2 Utilized Dataset

The quality of ECG signals provided by most of the portable ECG measuring instruments is very unstable and may include transient noises. Furthermore, the signals transmitted through wireless communication systems also exhibit even more unstable waveforms. These transient effects appear in the resulting feature vectors, hence they negatively affect the prediction accuracy of the subsequent ML method. In order to eliminate and smooth out these transient terms, we use the concept of *segmentation* here, where each segment includes multiple cardiac cycles, as shown in Fig. 2.1. Note that the number of cardiac cycles within each segment is a modeling parameter shown by s_w . Also, we can arbitrarily slide segments



Figure 2.1: Illustration of the segmentation stage. Each segment consists of $s_w = 3$ cardiac cycles. We slide segments with $n_s = 1$ to obtain the next segment. Each segment is converted to a feature vector representing a data sample.

equivalent to n_s cycles to generate the new feature vector. The parameters s_w and n_s can be tuned to improve the overall performance of the method. We choose $s_w = 3$ and $n_s = 3$, which yield the best classification accuracy, base on the intensive simulations. In Fig. 2.1, we have $s_{window} = 3$ and $n_{steps} = 1$.

For the purpose of training and evaluating classifier, MITDB is split into test (DS2) and training (DS1) sets by balancing the four classes according to [9].

The ECG signals in MITDB dataset are annotated and labels are provided for each cardiac cycle. However, we define a segment, which may include more than one cycle, as a sample. Hence, we need to translate per-cycle labels into per-segment labels. In this regard, a new label for each segment is generated by integrating all annotations of the beats within the segment. The segment is labeled as *normal*, if all member beats are annotated as N; otherwise, this segment will be labeled as the abnormality type of its member cycles if there is only one abnormality type. If more than one abnormality types are present within the segment, the segment is discarded. For instance, the segments with member cardiac

	Number of segments per AAMI class				
Evaluation Dataset	N	V	S	F	Total
DS1:Training	11721	2356	862	256	15195
DS2:Test	12633	2053	550	121	15357
Total	24354	4409	1412	377	30552

Table 2.1: Training and test datasets in MITDB.

cycles labeled as "NNN", "SSS", "NVV", "VVV" are respectively mapped to "N", "S", "V", and "V", and a segment with member cycels labeled as "NVS" is discarded. After segmentation and re-annotation, the total numbers of samples in the training and test set are obtained as summarized in Table 2.1.

2.3 ECG Signal Processing

2.3.1 Preprocessing

Biomedical signals, such as ECG signals, are composed of a sequence of signal segments, that can be presented by a set of statistical, morphological, and spectral features. Because of the time-varying properties of the signals, the traditional Fourier transform, which is unable to capture the time-varying statistics, is not suitable for this type of non-stationary signals. Wavelet decomposition solves this problem by scaling and translating the mother wavelet to constitute its basis functions, which capture both spectral and temporal properties of the signal. Given a time series, wavelet transformation decomposes the signal into a linear combination of basis functions. Thus, the basis functions with larger scales are smoother than those with smaller scales and consequently correspond to lower frequency components of the signal. Similarly, the decomposition coefficients correspond to higher frequencies when the scale is smaller. Using wavelet decomposition, we extract both time and frequency features of ECG signals. In this work, the wavelet analysis is applied to ECG signals in MITDB with a sampling frequency of 360 Hz. *Daubechies* wavelet of order 8 (db8) is selected as mother wavelet for denoising stage in this work for its optimal performance in increasing sign-to noise ratio [32]. The decomposition coefficients and their corresponding frequency components are presented in Fig. 2.2.

The low frequency noise or baseline wander between 0.15 to 1 Hz, caused by respiration and body movement, can be removed by deducting the approximation coefficient of level 8 (A_8) from the signal. The power of ECG signal is mainly concentrated in the frequency band spanning from 1 Hz to 40 Hz, hence higher frequency terms are more likely to represent noise terms including electromyogram induced noise and mechanical forces acting on the electrodes. These terms can be removed by discarding the detail coefficient of level 1 (D_1).

2.3.2 Segmentation

Most machine learning (ML) algorithms operate on input vectors and are not directly applicable to continuous signals. Therefore, biomedical signals are typically converted to a representative vector before incorporating to ML algorithms. We follow the common trend of translating a signal segment into a vector of representative features. In this regard, we first need to split the signal in time domain into smaller segments. To obtain more relevant results, we choose the segments as one or multiple consecutive cardiac samples, noting the fact that each cardiac cycles is associated with a label based on experts observation.

Most existing methods use wavelet analysis to detect the highest peak (R wave) in a cardiac cycle as a reference point and then use the signal morphology and typical properties of other waves to determine boundaries between consecutive intervals [20, 45–47]. In this study, we use this common approach as well. As was mentioned earlier in section 1.4.2, a cardiac



Figure 2.2: Frequency band of wavelet decomposition coefficients for MITDB signals.

cycle consists of five basic characteristic peaks: P, Q, R, S, and T. Among them, the QRS complex is the most significant peak in one cycle. The signal energy within one cardiac cycle is mainly concentrated within the QRS complex. The QRS complex also conveys important information that reflects the arrhythmia category [46]. An accurate detection of the QRS complex is of crucial importance for subsequent analysis. The energy of the QRS complex is generally within the range between 5 Hz and 25 Hz. For the ECG signals with sampling frequency of 360Hz, the QRS complex can be localized using the detail coefficients of level 5 (D_5) and level 6 (D_6)

The mother wavelet db4 is utilized at this stage due to its morphological similarity to QRS complexes [44]. By superimposing D_5 and D_6 , the information of QRS complex in the ECG signal can be characterized in a one-dimensional recombined signal ($QRS_DET = D_5 + D_6$). Other fiducial peaks (i.e. P, QRS onset, Q, S, QRS offset and T waves for each cardiac circle) are localized according to the algorithm proposed in [46]. The accuracy of peak detection and its coincidence with the true signal are shown in Fig. 2.3.

With the empirical values described in [46], we use 15% of the highest amplitude within the signal as the detection threshold. For most of the QRS complexes, their widths do not



Figure 2.3: (a): the detected R peak locations within the original ECG signal; (b): the corresponding R peak locations within the signal obtained by adding up the level 5 and level 6 detail coefficients of the wavelet decomposition of the signal $(QRS_DET = D_5 + D_6)$.

exceed 160ms, hence, we use a sliding window with a width of 160ms to detect the peaks in the QRS_DET. The window step size is set to 200ms, given that the time lag between the two adjacent cardiac cycles does not exceed 200ms [45]. Fig. 2.4 shows a typical QRS_DET waveform along with the corresponding 160ms window. The false peaks are eliminated using a 160ms time window, as seen in Fig. 2.4.

The T and P waves are outside the QRS window. Through scanning the region spanning from the end of the precedent QRS window to the beginning of the current QRS window, the P wave is located as the highest positive peak in this region. Similarly, the position of the T wave is obtained by finding the maxima of the signal in the region between the end of the QRS in the current cardiac cycle to the beginning of the next cardiac cycle. In an alternative



Figure 2.4: Window for detecting R peaks within QRS complexes.

way, the two peaks between the two consecutive QRS windows, respectively represent the T and P waves. As a result, an ECG signal in one cardiac cycle can be described by 7 fiducial points: P, QRS on, Q, R, S, QRS off and T.

After localizing the fiducial peaks, we can use the locations of R peaks to determine the boundaries of every cardiac cycle. By processing a large number of ECG signals, we realize that R peaks approximately split cardiac cycles into two pieces with 1/3 and 2/3 of the entire signal [46]. Based on this observation in the morphology of typical ECG signals, we define the starting point of a cardiac cycle as the point which divides the distance between the two consecutive R waves into two sections, where the length of the first section is half the length of the second section, as depicted in Fig. 2.1. In this figure, the average interval (RR) between the two adjacent R waves is denoted by h, therefore R wave is located in distance h/3 with respect to the beginning of the cycle. The advantage of this method is its low computational complexity and easy generalization to different patients, noting that a slight change in cardiac cycle boundaries does not significantly alter the properties of a cardiac cycle as long as T and P waves remain in the correct interval.

2.4 Feature Extraction

The feature extraction step plays a crucial role in the diagnosis of heart disease and has a great influence on the performance of the subsequent automated classification system. De Chazal *et al.* have studied the impact of using morphological features of ECG waveform on classification results [9]. As discussed in [20], the combination of three types of features (i.e. temporal, morphology, and spectral features) can provide a better discriminative power for the classification algorithm to distinguish between different types of arrhythmias.

According to several research works in the literature, different types of ECG samples significantly differ from each other in the power level of the frequency band between 5 Hz and 15 Hz [44]. Likewise, some other temporal features (such as the duration between the Q wave and the T wave, the P wave and the R wave, etc.) present different levels of correlation with the arrhythmia types of the sample [9]. Therefore, we choose to use the combination of temporal, morphological and spectral features as detailed in Table 2.2. Moreover, to account for the segment-level as well as the cycle-level characteristics, the extracted features include both cycle-based features (SET1) and segment-based features (SET2). SET1 includes the average and standard deviation of the corresponding features of the three cardiac cycles within a segment, and SET2 contains the overall characteristics of the time signal within a segment. Hence SET2 is calculated only once per segment. Therefore, we have a total of $8 \times 2 + 6 = 22$ features per segment as shown in Table 2.2. In other words, each feature vector is a 221 vector with zero-mean unit-variance elements after a proper normalization. In Table 2.2, mean($R_{i+1} - R_i$) refers to the mean of the time lag between two adjacent R waves

Feature Type	SET1	SET2		
Temporal Features	QRS duration	$\operatorname{mean}(R_{i+1} - R_i)$		
	QT duration	$\operatorname{mean}(R_i - R_{avg})$		
	PR duration			
Morphological Features	max positive peak to	signal average energy		
	second peak ratio	max positive peak		
		max negative peak		
		peak to energy ratio		
	signal power level at			
Spectral Features	7.5Hz, 10Hz, 12.5Hz,			
	15Hz			

Table 2.2: Features extracted from ECG signal

within a segment and mean $(R_i - R_{avg})$ is the mean of differences between the lengths of each cardiac cycle within the segment and the average cardiac cycle duration of the patient.

From Table 2.2, one may notice that these 22 features are not completely independent of each other. Also, some of the features may not be as relevant as the others. Therefore, we reduce the number of features to obtain a more robust predictive modeling [9,14]. We use Principal Component Analysis (PCA) as a commonly used the common dimension reduction method instead of explicit feature selection for its improved performance in biomedical signal processing [48,49]. We keep the 8 dominant directions of the signal after PCA as the most informative 8 features.

2.5 Classification Framework

In this section, we elaborate on the details of the proposed methodology to perform the classification and prediction tasks using the preprocessed ECG data. Based on our previous study [30] as well as the similar prior works on developing patient-specific classifiers [12–14], we propose a two-staged structure which includes a *global classifier* to capture general

properties of different classes followed by a *personalized classifier* to capture patient-specific properties [12–14, 30]. Moreover, the proposed algorithm incorporates a novel deviation analysis module with details presented in section 2.5.2.

2.5.1 Global Classifier

The flowchart in Fig. 2.5 presents the overall structure of the proposed system. As the first step of classification, the *global classifier* is trained using the whole training dataset. It facilitates the subsequent analysis in the system by identifying samples with severe morbidities. Depending on the application (properties of signals, the utilized labels, and the choice of features), different classification algorithms can be utilized [12, 14] for this step. Two important considerations include the classification accuracy and the computation complexity of the method [14]. Any abnormal label generated by the *global classifier* is considered as a *red alarm* and does not require further processing. However, the samples labeled as *normal* go through the subsequent *personalized classification* step.

2.5.2 Deviation Detection

One objective of this study is to identify the fuzzy states between normality and abnormalities. Therefore, the subsequent analysis focuses on processing samples classified as *normal* (N) by distinguishing whether or not they show considerable deviations towards one of the abnormality classes. For this purpose, an one-layer classifier is not sufficient due to multiple reasons: i) the numbers of samples in the normal and different morbid classes are not balanced in the training set DS1 as shown in Table 2.1, ii) the patient-specific normal trend is unknown, iii) detecting *yellow alarms* requires a new set of decision rules to determine whether or not the deviations are worthy of calling a yellow alarm. Therefore, a deviation


Figure 2.5: The general flowchart of proposed framework.

detection module is added after the global classifier to identify fuzzy states between normality and abnormalities using patient-specific normal cluster. In order to develop a ground for patient-specific normal functionality and adapt the classifier accordingly, the normal samples within first 5 minutes' signal of each patient are selected as the initialization of personalized dynamic normal cluster \mathcal{N}_0^k . As we collect more samples from the patient, the samples which are newly classified as normal (N) will be used to update the personalized dynamic normal cluster. Therefore, if a sample $\mathbf{x}_k(i)$ is classified as N after the deviation detection stage and the mahalanobis distance between $\mathbf{x}_k(i)$ and the centroid of the personalized dynamic normal cluster \mathbf{c}_N^k is less than 2, then $\mathbf{x}_k(i)$ is included in the updated \mathcal{N}_i^k . Mahalanobis distance quantifies the numbers of standard deviations between $\mathbf{x}_k(i)$ and \mathbf{c}_N^k , it can be formulated as follows:

$$d_{\text{mahal}} = \sqrt{(\mathbf{x}_k(i) - \mathbf{c}_N^k)' \mathbf{S}^{-1}(\mathbf{x}_k(i) - \mathbf{c}_N^k)}, \qquad (2.1)$$

where \mathbf{S}^{-1} stands for the covariance matrix of \mathcal{N}_i^k .

To distinguish between normal state and fuzzy states, we use a binary classification of N versus non-N, where the second includes all abnormal classes. We firstly calculate the following distance metrics:

$$R_i^{\max} = \max_{\mathbf{x}_j \in \mathcal{N}_i^k, \mathbf{x}_k \in \mathcal{N}_i^k} \{ \sqrt{(\mathbf{x}_j - \mathbf{x}_k)^2} \},$$
(2.2)

$$D_{\mathcal{X}}(\mathbf{x}_k(i)) = \underset{\mathbf{x}\in\mathcal{X}}{\operatorname{median}} \{ \sqrt{(\mathbf{x}_k(i) - \mathbf{x})^2} \},$$
(2.3)

$$D_{\mathcal{N}}^{\max}(\mathbf{x}_k(i)) = \max_{\mathbf{x}\in\mathcal{N}_i^k} \{\sqrt{(\mathbf{x}_k(i) - \mathbf{x})^2}\}.$$
(2.4)

The following conditions in Eq. 2.5 are then examined to verify if the deviation of a sample is within the range defined by α . Since some rare abnormalities are unlikely to be observed within the limited initialization period, the abnormal clusters $(\mathcal{S}, \mathcal{V}, \mathcal{F})$, which include abnormal beats from all patients in DS1, are deployed as the training dataset when calculating $D_{\mathcal{X}}(\mathbf{x}_k(i))$ in Eq. 2.3



Figure 2.6: The deviation analysis boundary restricts on fuzzy states between normality and abnormalities compared with the *global classifier* boundary.

$$\begin{cases} D_{\mathcal{N}}^{\max}(\mathbf{x}_{k}(i)) \leq \alpha R_{i}^{\max}, \\ D_{\mathcal{N}}(\mathbf{x}_{k}(i)) < D_{\mathcal{X}}(\mathbf{x}_{k}(i)) & \text{for } \mathcal{X} \in \{\mathcal{S}, \mathcal{V}, \mathcal{F}\} \end{cases}$$
(2.5)

If a sample labeled as *normal* by the *global classifier*, is again confirmed as N in this module, it will be used to update the \mathcal{N}_i^k as mentioned above. Otherwise, the system assumes that the sample has a considerable deviation towards one of the abnormal clusters and hence will pass it to the subsequent *personalized classifier*. The *personalized classifier* uses controlled transformation with optimized parameters to discern the deviation to different morbid types regardless of the cluster topology within the original feature space, as detailed in chapter 3 and chapter 4.

After performing both global and personalized classification steps, a given sample $x_k(i)$ of patient *i* at time *k* is mapped to a label $\hat{y}_k \in \{N, V_y, S_y, F_y, V_r, S_r, F_r\}$, where *N* stands for the *normal* state, and X_y and X_r stand for *yellow* and *red* alarms of type *X*.

2.6 Personalized Classifier

In this section, we provide the core idea behind the design of the *personalized classifier*, and the details of implementation will be discussed in chapter 3 and chapter 4. Since the proposed system aims at predicting the subsequent abnormality by analyzing a sample's deviation from the patient's normal functionality, it is vital to quantify the deviations using topological characteristics of the training data in the feature space. For most of the ECG applications, ECG signals are analyzed by their representative feature vectors. A natural choice for deviation analysis in the high-dimensional feature space is *cosine distance* (as defined in Eq. 2.6), which quantifies the distance between two vectors \mathbf{v} and \mathbf{w} .

$$d(\mathbf{v}, \mathbf{w}) = 1 - \frac{\mathbf{v}^T \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|} = 1 - \frac{\mathbf{v}^T \mathbf{w}}{\sqrt{(\mathbf{v}^T \mathbf{v})(\mathbf{w}^T \mathbf{w})}}$$
(2.6)

Consequently, relative deviations of a sample from normal cluster (\mathcal{N}_i^k) to other abnormal clusters $(\mathcal{S}, \mathcal{V}, \mathcal{F})$ are defined by the cosine distance between the vector $\mathbf{v}_k(i)$ (defined in Eq. 2.7) and the three vectors $\mathbf{v}_{\mathcal{X}}(i) = \mathbf{c}_{\mathcal{X}} - \mathbf{x}_k(i)$, where $\mathcal{X} \in \{\mathcal{S}, \mathcal{V}, \mathcal{F}\}$. In this case, a smaller cosine distance represents a higher alignment between the vector from the normal cluster centroid $\mathbf{v}_k(i)$ to the current sample $\mathbf{x}_k(i)$ and the reference vector from the normal cluster centroid to abnormal centroids $\mathbf{c}_{\mathcal{X}}$.

$$\mathbf{v}_{k}(i) = \mathbf{x}_{k}(i) - \mathbf{c}_{N}^{k}(i) = \mathbf{x}_{k}(i) - \sum_{\mathbf{x} \in \mathcal{N}_{i}^{k}} \mathbf{x}/|\mathcal{N}_{i}^{k}|, \qquad (2.7)$$

Therefore, the classification result of the *personalized classifier* $\hat{y}_k^2(i)$ is determined as follows:

$$\hat{y}_k^2(i) = \operatorname*{argmin}_{\mathcal{X} \in \{\mathcal{S}, \mathcal{V}, \mathcal{F}\}} \{ d(\mathbf{v}_k(i), \mathbf{v}_{\mathcal{X}}(i)) \}$$
(2.8)

The relevance of cosine distance depends on the topology of clusters in the feature space. The topology in feature space, itself is inherited from the feature extraction and feature selection methods. For example, as shown in Fig. 2.7, in the original feature space the overlaps and alignments of abnormal clusters may lead to inaccurate results of deviation detection. In order to eliminate the deviation analysis errors that arise from the asymmetry in the topology of clusters, it is desired to transform the original topology into a more symmetric one, where cosine distances directly reflect the amount of deviations.



Figure 2.7: Left: illustration of the cluster topology in the original feature space; Right: illustration of the cluster topology in the transformed feature space using a nonlinear mapping function.

For this purpose, two different spatial transformation methods are proposed in the next chapter 3 and chapter 4.

Chapter 3

Kernel-Based Nonlinear Spatial Transformation

3.1 Introduction

As discussed in chapter 2, the main objective of the *personalized classifier* is to reassess the normal samples to identify deviation of seemingly normal samples into any of the abnormality types. The original geometry of clusters in the feature space Ω^d depends on the choice of features implied by the feature extraction and feature selection stages g(). We noticed that with the resulting features in this work, the cluster geometry does no exhibit the necessary symmetric property and thus leads to a poor performance and even failure in predicting subsequent abnormalities. Therefore, an optimization method based on spatial transformation is proposed to solve this issue. More specifically, we propose a method to reshape the clusters such that

• the abnormal clusters surround the normal cluster;

- a maximal separation among the abnormal clusters are achieved;
- the angles between the vectors connecting the centroid of normal cluster to different abnormal clusters are equalized.

These properties can be achieved through imposing the following conditions:

- the vectors pointing from the normal centroid to different abnormal centroids present maximum mutual cosine distances;
- the overlapping parts among all clusters are minimized.

Given that the clusters in the original feature space do not meet these symmetric properties, developing a spatial transformation is unavoidable. In this chapter, a *kernel*-based nonlinear spatial transformation is proposed to reshape the feature space to reach the above-mentioned required symmetric properties. This reshaping process is part of the *personalized classification* stage (as shown in Fig. 2.5) of the ECG classification system as described in chapter 2. The nonlinear mapping projects the corresponding feature vector of each sample \mathbf{x}_k in the original space Ω^d onto a new vector \mathbf{z}_k in a higher dimensional space denoted by $\Omega^{d'}$. This is achieved using a nonlinear mapping function $\Psi^{d'}: \Omega^d \to \Omega^{d'}$. The resulting vectors are used by the *personalized classifier* to identify the minor *yellow alarm* type out of $\{\mathcal{S}, \mathcal{V}, \mathcal{F}\}$.

3.2 Kernel Method

Kernel method has been widely used in machine learning algorithms. For instance, it is the integral part of nonlinear support vector machine (SVM), which has been utilized in numerous applications recently [50]. Nonlinear kernel methods can efficiently improve the classification performance when there exists a nonlinear relationship between the input and output variables. Because of the complexity and diversity of feature vectors used in ECG analysis, the assumption of nonlinear relationship is considered valid in this work. Therefore, incorporating the nonlinear kernel method in the ECG analysis system can be beneficial.

In kernel SVM, the nonlinearities are introduced to the model through a kernel function, which implicitly maps data points $\mathbf{x_i} \in \mathcal{X}$ in the input space Ω into a Hilbert space Φ via a nonlinear function $\Psi()$ [51]. Then the algorithm minimizes the expected error E[L(y, f(x)]between the true labels y and the predicted labels f(x) for samples in a training dataset, by finding an optimal classification function f(), which also depends on the choice of $\Psi()$. L() here is an arbitrary loss function. A popular choice for L() is the least squared errors $\sum_{x_i \in \mathcal{X}} (y_i - f(x_i))^2$ [52]. Other choices for loss function include Hinge loss, absolute loss, hit and miss loss, etc. [53, 54].

If there are *m* observations in the input space, we use notation \mathbb{N} to represent the set of index (i.e. integers from 1 to *m*). Based on the input space $\mathbf{x_i} \in \Omega(i \in \mathbb{N})$ and classification mapping function *f*, the optimization problem can be written as:

minimize
$$\frac{1}{m} \sum_{i \in \mathbb{N}} L(y_i, f(\mathbf{x_i})) + \gamma ||f||^2,$$
 (3.1)

where $||f||^2$ is the squared norm of f. For instance, if f is a polynomial function of order p, (i.e. $f(v) = \alpha_0 + \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_i v_1^2 + \alpha_j v_1 v_2 + ... + \alpha_k v_N^p$), the norm of f is defined as:

$$||f||_2 = \sqrt{\sum_{j=0}^k \alpha_j^2}.$$
 (3.2)

The positive constant γ , also known as the regularization parameter, controls the balance between training error and the model complexity (smoothness). When optimizing the objective function, SVM only requires to calculate the inner products of the transformed features $\Psi(\mathbf{x})$ in the Hilbert Space Φ . Therefore, a kernel defined as $k(\mathbf{x_i}, \mathbf{x_j}) = \Psi(\mathbf{x_i})^T \Psi(\mathbf{x_j})$ can efficiently substitute the inner product calculation and induce the necessary nonlinearities into the model [55].

Different kernels represent different nonlinear mapping functions. For ML models, the selection of kernel plays a crucial role. Hence, there is no straightforward method to choose the best kernel and it is typically chosen by empirical approach and other heuristic model selection methods. An effective kernel function generally needs to satisfy the Mercer's condition, so that the inner products can be replaced by kernel functions, as used in SVM [56]. The exhaustive search for all possible kernels is a computationally expensive and unrealistic task [57]. A more efficient way to resolve this issue would be to search for an optimally weighted combination of a set of base kernels, such as polynomial kernel functions and Gaussian kernel functions [58]. This method has been proven to be robust and efficient since the base kernels satisfy Mercers condition individually and this property is consistent for different datasets [59].

The polynomial kernels are usually applied on normalized data for its explicit expression and steady performance. The polynomial kernels with lower degrees are more commonly used, since higher degrees tend to overfit the training data [60].

The Gaussian kernel function, denoted by $k(v, w) = exp(-\frac{||v-w||^2}{\sigma})$ for vectors v and w, is a very classic robust radial function, which has shown a high robustness in the case of noisy datasets [61]. However, it is equivalent to the inner product of samples after projecting into an infinite dimensional space. Therefore it is difficult to visualize the projected observations $\Psi(\mathbf{x})$ and interpret the results.

Considering the above-mentioned facts, the polynomial kernel is selected in this work for

the purpose of validating the proposed method and interpreting the effect of an optimized nonlinear kernel method on feature space reshaping. However, the proposed methodology is general and applicable to other nonlinear kernels.

The mapping function, which is a weighted combination of polynomial kernels can be explicitly written in the following format:

$$\mathbf{z}_{k} = \boldsymbol{\Psi}_{\mathbf{w}}(\mathbf{x}_{k}) = \begin{bmatrix} w_{1} \\ w_{2} \\ \vdots \\ w_{d'} \end{bmatrix} \circ \begin{bmatrix} \psi_{1}(\mathbf{x}_{k}) \\ \psi_{2}(\mathbf{x}_{k}) \\ \vdots \\ \psi_{d'}(\mathbf{x}_{k}) \end{bmatrix}, \qquad (3.3)$$

where \mathbf{w} is the vector of normalized coefficient.

Instead of selecting kernel, the process of spatial geometry optimization is accomplished by adjusting the coefficients of fixed polynomial basis functions $\psi()$. Since the number of free parameters increases exponentially with the order of polynomial kernel, an exhaustive search is not practical for parameter optimization. Therefore, it is necessary to implement a heuristic optimization algorithm, in which parameters are obtained by maximizing the designed objective functions. More specifically, the nonlinear reshaping module in this chapter aims to adjust mapping coefficients $\mathbf{w} = [w_1, w_2, \dots, w_d]^T$ to achieve the ideal symmetric geometry in the reshaped feature space while maintaining the maximal separation between clusters.

3.3 Multi-objective Optimization

3.3.1 Objective Functions

To elucidate the details of the optimization problem, here we consider an illustrative example, where the original feature space is a 2-dimensional space Ω^2 . We also assume for simplicity that the order of the polynomial function is 2. Therefore, we have:

$$\mathbf{x} = [x_1 \ x_2]^T, \quad \mathbf{w} = [w_1 \ w_2 \ \dots \ w_5]^T, \quad d = 2, \quad d' = 5,$$

$$\psi_1(\mathbf{x}) = x_1, \psi_2(\mathbf{x}) = x_2, \psi_3(\mathbf{x}) = x_1^2, \psi_4(\mathbf{x}) = x_2^2, \psi_5(\mathbf{x}) = x_1 x_2. \tag{3.4}$$

The coefficient vector \mathbf{w} is a 5 × 1 vector. Therefore, the algorithm needs to search in a 5-dimensional space to find an optimal vector \mathbf{w} so that the geometry of feature space after transformation has the desired properties described in section 3.1. For this purpose, we define two objective functions to impose the symmetry and separation of different abnormality clusters. The concepts of *Fisher discriminant* function and cosine distance are used to quantify these properties. The objective functions are formulated as follows:

$$o_{1}(\mathbf{w}) = \frac{1}{\min_{\substack{c,d=2,\dots,p \text{ and } c\neq d}} \{d(\mathbf{v}_{\mathcal{X}_{c}}, \mathbf{v}_{\mathcal{X}_{d}})\}}$$
(3.5)
$$o_{2}(\mathbf{w}) = \frac{SW}{SB} = \frac{\sum_{c=1}^{C} \sum_{\mathbf{z}\in\mathcal{X}_{c}} (\mathbf{z} - \mathbf{c}_{\mathcal{X}_{c}})^{T} (\mathbf{z} - \mathbf{c}_{\mathcal{X}_{c}})}{\sum_{c=1}^{C} \sum_{d=1, d\neq c}^{C} (\mathbf{c}_{\mathcal{X}_{c}} - \mathbf{c}_{\mathcal{X}_{d}})^{T} (\mathbf{c}_{\mathcal{X}_{c}} - \mathbf{c}_{\mathcal{X}_{d}})}$$

The maximization of pairwise cosine distance between the vectors $\mathbf{v}_{\mathcal{X}_c}$, $\mathbf{v}_{\mathcal{X}_d}$ connecting the centroid of the normal cluster to the centroids of abnormal clusters \mathcal{X}_c and \mathcal{X}_d is achieved by

minimizing $o_1(\mathbf{w})$. In fact, this objective function is deduced from discrimination function of *personalized classifier* in Eq. 2.8. Cosine distance is defined by Eq. 2.6 and the calculation of $\mathbf{v}_{\mathcal{X}_{c,d}}$ can be written as follows:

$$\mathbf{v}_{\mathcal{X}_i} = \mathbf{c}_N^k - \mathbf{c}_{\mathcal{X}_i} \tag{3.6}$$

Since for some patients, the total number of a certain type of abnormal samples is very limited, the abnormal samples in training set DS1 are utilized in calculating the two objective functions. In Eq. 3.5, the abnormal cluster centroids are calculated using the abnormal samples in training dataset DS1, while the centroid of the normal cluster is defined by the preceding normal samples for the same person.

On the other hand, $o_2(\mathbf{w})$ represents the ratio of the within-cluster variance to the betweencluster variance and consequently controls the separation between the clusters. By minimizing $o_1(\mathbf{w})$ and $o_2(\mathbf{w})$ jointly, the algorithm eliminates the ambiguity of classification while improving the predictive power of the *personalized classifier* due to the symmetric geometry of clusters.

3.3.2 Multi-objective Particle Swarm Optimization

We notice that $o_1(\mathbf{w})$ and $o_2(\mathbf{w})$ are not necessarily independent of each other. Thus, the optimization problem defined above is equivalent to joint minimization of $o_1(\mathbf{w})$ and $o_2(\mathbf{w})$ subject to a constraint condition: $|\mathbf{w}|_2 = 1$. This constraint is necessary since the first objective function $o_1(\mathbf{w})$ is inversely proposal to $|\mathbf{w}|_2$, whereas the second objective function $o_2(\mathbf{w})$ is scale-invariant.

This problem is a non-convex multi-objective optimization problem. Therefore, neither closed form solutions nor the optimization methods proposed for convex problems are applicable to this case. In this work, we utilize the multi-objective particle swarm optimization (MOPSO) algorithm to solve this optimization problem and obtain the optimal coefficients [62].

Particle swarm optimization (PSO) is a heuristic searching algorithm and has the advantage of fast convergence, and easy implementation [62, 63]. PSO is defined to solve problems with a single objective function, where closed form solutions are not tractable. Several research works are devoted in the past decade to extend this method to multi-objective optimization problems [64, 65]. In the MOPSO framework, the goal is to solve the typical *Pareto optimization problem* based on the evolutionary algorithm used in PSO. In other words, it aims at solving an optimization problem with two or more conflicting objective functions by approximating the *Pareto front*.

In order to compare different set of coefficients in this optimization problem, the concept of Pareto front is briefly introduced in this section. For a multi-objective optimization problem with two objective functions, if a solution \mathbf{w}^1 is said to *dominate* another \mathbf{w}^2 , the following two conditions are satisfied:

- 1. $o_1(\mathbf{w}^1) \le o_1(\mathbf{w}^2)$ and $o_2(\mathbf{w}^1) \le o_2(\mathbf{w}^2)$
- 2. $o_1(\mathbf{w}^1) < o_1(\mathbf{w}^2)$ or $o_2(\mathbf{w}^1) < o_2(\mathbf{w}^2)$

If a solution is not dominated by any other solution in the searching space, then this solution is an *optimal* solution for this problem. A Pareto front is defined by the set of Pareto optimal solutions. However, in non-convex optimization, the Pareto front can not be represented explicitly by a deterministic function. Therefore, the majority of the optimization methods use heuristic searching algorithms to approximate the Pareto front [62].



Figure 3.1: Particles stored in external repository approximate the Pareto front.

Implementation Details of MOPSO

MOPSO is a popular multi-objective optimization algorithm, which uses Pareto dominance to determine the direction of a particle in each iteration by combining the optimal direction found by the particle itself and the global optimal direction. Among different implementations of MOPSO, the algorithm proposed by Coello Coello and Lechug presents a better performance and lower computational complexity in most applications [62]. Therefore, this algorithm is implemented and utilized in this work to solve the multi-objective optimization problem. One special property of this algorithm is the use of external repository, in which all Pareto optimal particles for every swarm are recorded for each iteration. The solution represented by repository members are stored and used as an optimal approximation of the Pareto front because they converge to the actual Pareto front as proved in [62].

Fig. 3.1 presents the results of joint minimization of objective functions $o_1(\mathbf{w})$ and $o_2(\mathbf{w})$.

This figure demonstrates that the repository members are Pareto optimal compared to the other particles. This figure also confirms that the repository members converge to a uniform Pareto front.

Using the concept of Pareto optimality, we demonstrate the impact of applying kernel functions in this spatial reshaping problem by comparing the Pareto front of the optimization problem obtained by using MOPSO for two different scenarios including i) the optimized coefficients for the linear combination of data in the original feature space (i.e. a linear identity function) and ii) the transformed samples under polynomial kernel whose coefficients are optimized using MOPSO. Therefore, we first optimize the coefficients of the third-order polynomial kernel function, as formulated in Eq. 3.4 and then optimize the coefficients of linear features in the origin feature space. The purpose of this comparison is to investigate whether or not the objective functions are fundamentally improved by incorporating nonlinear terms into the feature vectors through the proposed polynomial function.

As shown in Fig. 3.2, the estimated Pareto front of the nonlinear model using the polynomial kernel dominates the Pareto front of the original linear model. This result is expected since the transformed samples exhibit a higher degree of freedom by adding new dimensions to the data in the feature space through the nonlinear mapping. A higher degree of freedom enables the MOPSO algorithm to tune the optimization parameters and find better solutions than the best achievable solutions by the original data samples. In other words, the kernel method combined with multi-objective particle swarm optimization algorithm can improve the spatial topology of the clusters quantified by the two objective functions.



Figure 3.2: The Pareto front approximated by MOPSO is significantly shifted when using the transformed feature vectors. This improvement is due to the increase in the degree of freedom provided by additional non-linear dimensions added to the samples.

3.4 Experimental Results

As mentioned in section 2.3, a sample segment is represented by an 8-dimensional vector $\mathbf{x} = [x_1, x_2, \dots, x_8]$ after the feature extraction stage and the PCA-based dimension reduction stage. To specify the nonlinear transformation in Eq. 3.3, a polynomial function of order 3 is applied to the feature vectors. The resulting transformed vectors are as follows:

$$[x_1, x_2, \dots, x_8, x_1^2, x_2^2, \dots, x_8^2, x_1^3, x_2^3, \dots, x_8^3, x_1x_2, \dots, x_6x_7x_8],$$
(3.7)

which include 165 terms and 8 of them are the original features. This high dimensionality may cause the classifier to be trapped into the overfitting problem. It also significantly increases the computational complexity of the algorithm. To solve this issue, we discard some of the induced terms and include only 8 square terms x_i^2 , 8 cubic terms x_i^3 , and 8 cross terms of power two $x_i x_j$ and 8 cross terms of power three $x_i x_j^2$. We randomly choose these terms after discarding the redundant cross terms. Therefore, the mapped vectors $\mathbf{z}_{32\times 1}$ include a total of 32 terms as follows:

$$\mathbf{z} = \{x_i^2 | i = 1, 2 \dots 8\} \cup \{x_i^3 | i = 1, 2 \dots 8\} \cup \{x_i x_j | i, j = 1, 2 \dots 8, i \neq j\} \cup \{x_i^2 x_j | i, j = 1, 2 \dots 8, i \neq j\}$$
(3.8)

The performance of the aforementioned *kernel*-based method is tested on DS2 excluding record 232, for this record has only 7 normal samples. In total, 21 records are used to evaluate the performance.

Table 3.1 shows the performance of the proposed method in classifying ECG signal segments. In order to evaluate the consistency as well as the general classification results over all recordings, the median, interquartile range (IQR), mean and standard deviation of accuracy (AC), sensitivity (SE) and specificity (SP) are presented. The results are promising and the median of the classification accuracy for all classes are in the range of 88% - 99%. Sensitivity and specificity of the proposed method exhibit similar ranges. The mean accuracy is at least 86% except for class V. Therefore, this system is not likely to miss important alarms or to report false alarms.

More importantly, the predictive capability of the proposed method is worthy of evaluating, since it is an unique feature provided by the proposed system. In order to quantify the posterior probability of observing an abnormal signal after a preceding *yellow alarm* of the same type in Eq. 2.8, the number of predicted samples are counted as formulated in Eq. 3.9:

Class N	median(%)	IQR(%)	$\operatorname{mean}(\%)$	std (%)
AC	94.8	19.52	86.62	18.55
SE	97.21	17.36	87.47	19.26
class V	median(%)	IQR(%)	$\operatorname{mean}(\%)$	std (%)
AC	86.11	27.54	76.41	22.81
SP	99.71	11.22	90.18	18.52
class S	median(%)	IQR(%)	$\operatorname{mean}(\%)$	std (%)
AC	99.28	2.24	98.29	2.57
SP	99.64	22.17	97.56	6.06
class F	median(%)	IQR(%)	$\operatorname{mean}(\%)$	std (%)
AC	97.91	8.2	93.85	7.84
SP	100.00	0.03	99.12	3.6

Table 3.1: Classification results of the proposed method.

$$P(\hat{y}_{k+i} = X_r | \hat{y}_k = X_y) = \frac{\# \text{ of } y_{k+i} = X \text{ after } \hat{y}_k = X_y}{\# \text{ of true alarms after } \hat{y}_k = X_y}$$

$$P(\hat{y}_{k+i} = X_r) = \frac{\# \text{ of true alarm of type } X (y_k = X)}{\# \text{ of all true alarms}}$$
(3.9)

The summary of results for all 21 test records is presented in Table 3.2. The values provided under the column *Probability of subsequent abnormality* (%) in Table 3.2 represent the probabilities of having a subsequent true abnormality of all types after observing a *yellow alarm* of all types along with the prior probabilities of observing a certain abnormal type regardless of the preceding *yellow alarm* typt in the very last column. These results confirm the predictive capability of *yellow alarms* as well as the scientific fact that the *yellow alarms* are indicative of upcoming red alarms. This conjecture supported by the fact that at least some of the heart problems develop over time, although the symptoms may appear suddenly.

For instance, the prior probabilities of observing a sample segment with abnormal types V, S, and F are respectively $\frac{96}{96+29+18} = 67\%$, $\frac{29}{96+29+18} = 20\%$ and $\frac{18}{96+29+18} = 13\%$, based on their relative frequencies in the dataset. However, the corresponding posterior probabilities after

ving a	red alarm of the	same type.	
		Count numbers of sub-	Probability of subse-
		sequent abnormality	quent abnormality (%)

 V_y

75

21

4

Total

96

29

18

 S_y

75

29

6

 F_y

61

14

25

Total

67

20

13

 F_y

35

8

14

Table 3.2: Predictive power of *yellow alarms*: A *yellow alarm* increases the chance of observing a red alarm of the same type.

observing a yellow alarm of type V_y , S_y , F_y are respectively $\frac{38}{38+11+2} = 75\%$, $\frac{10}{23+10+2} = 29\%$
and $\frac{14}{35+8+14} = 25\%$. This means that the probability of observing a real abnormal segment
of type V is $75\% - 67\% = 8\%$ higher than its prior probability. The same trend holds
for other types of <i>yellow alarm</i> as well. The results suggest a more in-depth study of the
concept of <i>yellow alarms</i> for heart monitoring. We conclude this section by stating that a
new methodology provided in chapter 4 to optimize the nonlinear transformation using an
analytical approach, which significantly reduces the computational cost.

3.5 Summary of Contributions

 V_y

38

11

2

yellow alarm

True V

True S

True F

 S_y

23

10

2

In this chapter, we proposed a novel method which combines kernel-based nonlinear transformation with MOPSO optimization method. With the concept of kernel method and the loss function, we implemented a novel method using a weighted combination of base nonlinear kernels to reshape the input feature space by mapping it to a high-dimensional space. The coefficients of kernels are optimized according to two conditions, namely, maximum separation between the clusters and maximum cosine similarities between the abnormal clusters. By studying kernel methods as well as heuristic optimization methods based on the spatial topology optimization concept, the outcome of this chapter can be concluded as follows:

- translate the ideal symmetric geometry of feature space into two objective functions;
- design a kernel-based transformation using weighted combination of base kernels;
- validate that nonlinear transformations are more capable of improving the spatial topology than linear transformations.

In order to verify the efficiency of this method, we applied the developped method to test samples in the MIT-BIH ECG recording database [28]. The experimental results show that the proposed method has a classification accuracy in the range of 88% - 99% for different ECG records.

Furthermore, the proposed algorithm demonstrates the potential of assessing a signal's deviation from its patient-specific normal trend in order to indicate the upcoming abnormality classes. The predictive capacities of the system is tested with ECG signals, but the proposed method is general and applicable to similar problems beyond the ECG signal processing, studied in this work. If a biomedical signal has one base class (i.e. normal state) and several abnormal states, we can deploy the proposed method to trigger *yellow alarms* and predict upcoming abnormality classes.

Chapter 4

Controlled Spatial Transformation With Deterministic Mapping Function

4.1 Introduction

In chapter 2, we discussed the details of the patient-adaptive ECG classification framework. An important part of this methodology is to design a *personalized classifier* with a *deviation analysis module*. The performance of the *deviation analysis module* depends on the geometry of different clusters of data samples in the feature space. In chapter 3, we proposed a novel spatial transformation method based on MOPSO to achieve a desired symmetry in the transformed feature space to boost the performance of the deviation analysis module. This methodology enables us to further process the normal samples and identify fuzzy states between the normal and abnormal states. However, the proposed method uses an iterative optimization which may have high computational complexity. In this chapter, we introduce another deterministic spatial transformation method to model the fuzzy state of samples with a more tractable and analytical solution.



Figure 4.1: Left: illustration of the clustering topology in the transformed feature space without reducing the within-cluster variance; Right: illustration of the clustering topology in the feature space transformed with the optimized mapping function, which reduces the within-cluster variance.

While the method presented in chapter 3 yields the desired capacity of predicting upcoming abnormalities, the interpretation of the mechanisms of the system is not straightforward and easily tractable, thus it is not easily generalizable to a broader range of applications in the biomedical signal processing. The main objective of this chapter is to develop a deterministic transformation with a stronger prediction power based on a controlled spatial topology studied in chapter 3. In this method, we further optimize the topology and spatial geometry of the clusters in the feature space by reducing the within-cluster variance after the spatial transformation. As shown in Fig. 4.1, through this improvement, the performance of the *personalized classifier* in terms of prediction accuracy can be further enhanced with the proposed method.

4.2 Hyper-Spherical Coordinates

In chapter 3, the spatial transformation module is implemented with a polynomial kernel function and a heuristic optimization method. The system performance has been proven to be promising. In addition to the general drawbacks of heuristic method, it is not straightforward to select an appropriate base kernel as the core of spatial mapping function due to the high variety of kernel functions.

In order to address this issue, a novel deterministic spatial mapping function is proposed in this chapter based on *hyper-spherical* coordinates [66]. Since hyper-spherical coordinates consist of angles and radius, these parameters in both original feature space and the desired target space are used to determine the mapping function.

The hyper-spherical coordinate system (*n*-dimensional spherical coordinate system) and its mapping to the *Cartesian* coordinate system are first introduced in [66]. If **x** is a sample vector in a *n*-dimensional feature space, with its Cartesian coordinates $(\xi_1, \xi_2 \dots \xi_n)$, then its corresponding hyper-spherical coordinates can be obtained through Eq. 4.1, which is originally derived through its reverse mapping (Eq. 4.2) using equation: $\sin(\arccos(x)) = \sqrt{1-x^2}$.

$$r = \sqrt{\xi_n^2 + \xi_{n-1}^2 + \dots + \xi_2^2 + \xi_1^2}$$

$$\theta_1 = \arccos \frac{\xi_1}{\sqrt{\xi_n^2 + \xi_{n-1}^2 + \dots + \xi_1^2}}$$

$$\theta_2 = \arccos \frac{\xi_2}{\sqrt{\xi_n^2 + \xi_{n-1}^2 + \dots + \xi_2^2}}$$

$$\vdots$$

$$\theta_{n-2} = \arccos \frac{\xi_{n-2}}{\sqrt{\xi_n^2 + \xi_{n-1}^2 + \xi_{n-2}^2}}$$

$$\theta_{n-1} = \begin{cases} \arccos \frac{\xi_{n-1}}{\sqrt{\xi_n^2 + \xi_{n-1}^2}} & \xi_n \ge 0\\ -\arccos \frac{\xi_{n-1}}{\sqrt{\xi_n^2 + \xi_{n-1}^2}} & \xi_n < 0 \end{cases}$$
(4.1)

$$\xi_{1} = r \cos(\theta_{1})$$

$$\xi_{2} = r \sin(\theta_{1}) \cos(\theta_{2})$$

$$\xi_{3} = r \sin(\theta_{1}) \sin(\theta_{2}) \cos(\theta_{3})$$

$$\vdots$$

$$\xi_{n-1} = r \sin(\theta_{1}) \cdots \sin(\theta_{n-2}) \cos(\theta_{n-1})$$

$$\xi_{n} = r \sin(\theta_{1}) \cdots \sin(\theta_{n-2}) \sin(\theta_{n-1}), \qquad (4.2)$$

where $0 \le \theta_j \le \pi$, $j = 1, ..., n - 2; 0 \le \theta_{n-1} \le 2\pi; 0 \le r < \infty$.

4.3 Orthogonalization

To simplify the algorithm, in this chapter, the topology of clusters in the feature space is approximated by their centroid locations, denoted by $\mathbf{c}_N^k, \mathbf{c}_V, \mathbf{c}_S, \mathbf{c}_F$, respectively for the normal cluster and the three abnormality clusters of type V, S, and F. Furthermore, as we assume that the samples with fuzzy states deviate from the normal cluster to a specific abnormal cluster, the spatial topology stays unchanged if the normal centroid is simply translated to the origin of the Cartesian coordinate system. The clustering topology in the original feature space can be equivalently represented by the following matrix with three row vectors:

$$\mathcal{C} = \begin{bmatrix} \mathbf{c}_{\mathcal{V}} - \mathbf{c}_{N}^{k} \\ \mathbf{c}_{\mathcal{S}} - \mathbf{c}_{N}^{k} \\ \mathbf{c}_{\mathcal{F}} - \mathbf{c}_{N}^{k} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{\mathcal{V}\mathcal{N}} \\ \mathbf{v}_{\mathcal{S}\mathcal{N}} \\ \mathbf{v}_{\mathcal{F}\mathcal{N}} \end{bmatrix}$$
(4.3)

As shown in Fig. 4.1, in order to improve the performance of the *personalized classifier* and to avoid ambiguity in quantifying the deviations to different abnormal classes, a topology with a maximum separation between the three vectors of C is preferred. To achieve a lower computational complexity in higher order dimensions, the algorithm aims at transforming vectors in C to orthogonal vectors using a deterministic function. This approach not only simplifies the transformation derivations, it also ensures a full symmetry among abnormality clusters. As such, the problem reduces to finding a transformation that maps a set of vectors to a set of orthogonal vectors in the same space. Therefore, we can use the popular *Gram-Schmidt* orthogonalization method explained in [67,68]. Hence, in the first step of this stage, the three row vectors of C representing the centroids of the three abnormality clusters are fed to the orthogonalization process as follows:

$$\mathcal{C}^{\perp} = \text{Gram-Schmidt}(\mathcal{C}) = \begin{bmatrix} \mathbf{v}_{\mathcal{V}\mathcal{N}}^{\perp} \\ \mathbf{v}_{\mathcal{S}\mathcal{N}}^{\perp} \\ \mathbf{v}_{\mathcal{F}\mathcal{N}}^{\perp}, \end{bmatrix}$$
(4.4)

where \mathcal{C}^{\perp} is the matrix of orthogonalized vectors in the Cartesian coordinate system. The hyper-spherical coordinates \mathcal{C}^{\perp}_{*} of these orthogonalized vectors are calculated subsequently using Eq. 4.1. After this step, the orthogonalized vectors in the hyper-spherical coordinate system are obtained as formulated in Eq. 4.5. The j^{th} angular dimension denoted as θ^{\perp}_{j} and the radius is noted as r^{\perp} .

$$\mathcal{C}_{*}^{\perp} = \begin{bmatrix} r_{\mathcal{V}\mathcal{N}}^{\perp} & \theta_{1_{\mathcal{V}\mathcal{N}}}^{\perp} & \dots & \theta_{n-1_{\mathcal{V}\mathcal{N}}}^{\perp} \\ r_{\mathcal{S}\mathcal{N}}^{\perp} & \theta_{1_{\mathcal{S}\mathcal{N}}}^{\perp} & \dots & \theta_{n-1_{\mathcal{S}\mathcal{N}}}^{\perp} \\ r_{\mathcal{F}\mathcal{N}}^{\perp} & \theta_{1_{\mathcal{F}\mathcal{N}}}^{\perp} & \dots & \theta_{n-1_{\mathcal{F}\mathcal{N}}}^{\perp} \end{bmatrix}$$
(4.5)

4.4 Spatial Mapping Function

After obtaining the original hyper-spherical coordinates of $[\mathbf{v}_{\mathcal{V}\mathcal{N}}, \mathbf{v}_{\mathcal{S}\mathcal{N}}, \mathbf{v}_{\mathcal{F}\mathcal{N}}]^T$ and the orthogonalized hyper-spherical coordinates $[\mathbf{v}_{\mathcal{V}\mathcal{N}}^{\perp}, \mathbf{v}_{\mathcal{S}\mathcal{N}}^{\perp}, \mathbf{v}_{\mathcal{F}\mathcal{N}}^{\perp}]^T$, the goal is to design a mapping function $\mathbf{F} : \mathbf{R}^n \to \mathbf{R}^n$ from the original coordinates to the orthogonal ones which exhibit the desired clustering topology, such that Eq. 4.6 holds.

In the Gram-Schmidt algorithm, the very first vector serves as a reference vector and remains unchanged in the orthogonalization process, namely $\mathbf{v}_{\mathcal{V}\mathcal{N}} = \mathbf{v}_{\mathcal{V}\mathcal{N}}^{\perp}$. Therefore the mapping function **F** shall satisfy the following equations:

$$\mathbf{F}(\mathbf{v}_{SN} - \mathbf{v}_{VN}) = \mathbf{v}_{SN}^{\perp} - \mathbf{v}_{VN}^{\perp} = \mathbf{v}_{SN}^{\perp} - \mathbf{v}_{VN}$$

$$\mathbf{F}(\mathbf{v}_{FN} - \mathbf{v}_{VN}) = \mathbf{v}_{FN}^{\perp} - \mathbf{v}_{VN}^{\perp} = \mathbf{v}_{FN}^{\perp} - \mathbf{v}_{VN}$$
(4.6)

Furthermore, since the orthogonality of vectors is independent to their radii r, we only need to design \mathbf{F} for the n-1 angular dimensions $(\theta_1, \ldots, \theta_{n-1})$, and the coordinate rremains unchanged after the mapping. Consequently, the function \mathbf{F} can be decomposed into n-1 functions: $f_i : \mathbf{R} \to \mathbf{R}, \ i = 1 \dots n-1$ with constraints in Eq. 4.6 as well as two additional extreme points defining the range of input and output values. We use the notation $\mathbf{v}_{SN} - \mathbf{v}_{VN} = \Delta_{SV}$ and denote the i^{th} angular dimension of Δ_{SV} as $\delta_{i_{SV}}$. We use similar notations for the other vector, (i.e. $\mathbf{v}_{\mathcal{FN}} - \mathbf{v}_{\mathcal{VN}}$). Hence for each angular dimension i, f_i is determined by $(\delta_{i_{\mathcal{SV}}}, \delta_{i_{\mathcal{SV}}}^{\perp})$ and $(\delta_{i_{\mathcal{FV}}}, \delta_{i_{\mathcal{FV}}}^{\perp})$, along with the two *extreme boundary points* defining the domain and range of the functions.

In order to maintain the simplicity and the linearity of the mapping function, the functions f_i should be continuous and monotonic. For this purpose, the valid range of angular dimensions (after considering the periodicity of these functions) is used to determine the boundary constrains; therefore the problem is transformed into a curve fitting problem.

For example, if linking $(\delta_{i_{SV}}, \delta_{i_{SV}}^{\perp})$ and $(\delta_{i_{FV}}, \delta_{i_{FV}}^{\perp})$ results in a monotonically increasing function, noting the fact that the range of n-2 first angular dimensions is the interval $[0, \pi]$, then the two extreme boundary points would be (0, 0) and (π, π) , as depicted in Fig. 4.2. Conversely, for a monotonically decreasing function, the extreme boundary points would be $(\pi, 0)$ and $(0, \pi)$. These rules apply for the functions defined for the n-2 first angular dimensions. Similar rules apply to the last angular dimension with only one consideration that the period is 2π instead of π . Since $(\delta_{i_{SV}}, \delta_{i_{SV}}^{\perp})$ and $(\delta_{i_{FV}}, \delta_{i_{FV}}^{\perp})$ represent the orthogonalization process, they are decisive for this process and will be called as *orthogonalization points*. So we call these two points as well as the two extreme boundary points as *target points* of the angular dimensions in the following sections.

The simplest candidate function for f_i , which connects all the four target points, including orthogonalization points and extreme boundary points, in the 2-D plane would be a linear spline as shown in Fig. 4.2.



Figure 4.2: The simple mapping function for one angular dimension which maps the target points in the original space to the desired target points. Target points include the two extreme boundary points (0,0), and (π,π) as well as $(\delta_{i_{SV}},\delta_{i_{SV}}^{\perp})$ and $(\delta_{i_{FV}},\delta_{i_{FV}}^{\perp})$ to yield the desired mapping.

4.5 Optimized Mapping Function

The mapping function in Fig. 4.2 exhibits the two desired properties of monotonicity and continuity for an ideal mapping function f_i . However, this simple piece-wise linear mapping function suffers from some drawbacks. Firstly, the function is not differentiable at the orthogonalization points $(\delta_{i_{SV}}, \delta_{i_{SV}}^{\perp})$ and $(\delta_{i_{FV}}, \delta_{i_{FV}}^{\perp})$, which can lead to severe cluster deformations. Secondly, the mapping function is applied to the angular dimensions, the discontinuity of the derivative of the function can cause the convex-clusters to be mapped into non-convex clusters in the Cartesian coordinate system. In order to avoid deformations and to preserve maximal similarity with the original clustering geometry, it is desired to use a more linear functions with continuous derivatives. Also, in order to provide maximal separation between clusters and to concentrate clusters into disjoint non-overlapping clusters, it is beneficial to map the regions between two target points into a region with equal or even smaller range in

the Cartesian coordinate system. In other words, it is desired to keep the samples close to the centroids in the mapped space as much as possible. This property is achieved by using nonlinear functions. However, there is a trade-off between the level of concentration and the linearity, which needs to be carefully addressed. In order to avoid deformation while providing maximal separation, an optimized mapping function is proposed in this section.

In order to accommodate the above-mentioned requirements, it is desired to find a function, which satisfies the following mathematical conditions:

- the function is differentiable everywhere (continuous first derivative);
- the derivative of the function is small at the target points, which correspond to the centroids of clusters in the original and transformed space, i.e. δ_{i_{SV}} and δ_{i_{FV}} in Fig. 4.3;
- the derivative of the function is large at the boundaries of two regions (point $(\epsilon_{i_{SV}}, \epsilon_{i_{SV}}^{\perp})$ in Fig. 4.3);

Therefore, we propose to use the basic function p with adjustable parameters, which satisfies the aforementioned conditions. Each target point is associated with a basic function. The function p is composed of two constitutive functions: h(x) and g(x), respectively defined in two regions: i) from the target point to the upper boundary point, ii) from the lower boundary point to the next target point. Before defining the two functions h(x) and g(x), we need to define the boundaries between two consecutive target points $(\delta_{i_{SV}}, \delta_{i_{SV}}^{\perp})$ and $(\delta_{i_{FV}}, \delta_{i_{FV}}^{\perp})$, in which these functions are defined. We simply choose the midpoint as the boundary points for simplicity. For instance, we have $\epsilon = (\delta_1 + \delta_2)/2$ as stated in Eq. 4.7. The lower and upper boundary points are noted respectively as (γ, γ^{\perp}) and $(\epsilon, \epsilon^{\perp})$. To ensure the continuity of the mapping function f, the boundaries and target points should satisfy:

$$(\epsilon_{i_{\mathcal{S}\mathcal{V}}}, \epsilon_{i_{\mathcal{S}\mathcal{V}}}^{\perp}) = (\gamma_{i_{\mathcal{F}\mathcal{V}}}, \gamma_{i_{\mathcal{F}\mathcal{V}}}^{\perp}) = (\frac{\delta_{i_{\mathcal{S}\mathcal{V}}} + \delta_{i_{\mathcal{F}\mathcal{V}}}}{2}, \frac{\delta_{i_{\mathcal{S}\mathcal{V}}}^{\perp} + \delta_{i_{\mathcal{F}\mathcal{V}}}^{\perp}}{2})$$
(4.7)

The two piece-wise functions $h(x; \alpha, \epsilon, \epsilon^{\perp}, \delta, \delta^{\perp})$ and $g(x; \alpha, \gamma, \gamma^{\perp}, \delta, \delta^{\perp})$ are developed using the inverse of logit function, hence they are defined as follows:

$$K_{h} = \frac{\epsilon^{\perp} - \delta^{\perp}}{e^{\alpha(\epsilon - \delta, 0)^{+}} - 1}$$

$$h(x; \alpha, \epsilon, \epsilon^{\perp}, \delta, \delta^{\perp}) = K_{h}[e^{\alpha(x - \delta, 0)^{+}} - 1] + \delta^{\perp}$$

$$(4.8)$$

$$K_g = \frac{\gamma^{\perp} - \delta^{\perp}}{e^{\alpha(-\gamma+\delta,0)^+} - 1}$$

$$g(x; \alpha, \gamma, \gamma^{\perp}, \delta, \delta^{\perp}) = K_g[e^{\alpha(\delta-x,0)^+} - 1] + \delta^{\perp}$$

$$(4.9)$$

In order to investigate the satisfaction of the above-mentioned conditions, we first verify that these functions pass through the target points. More specifically, g() should pass through points $(\gamma_i, \gamma_i^{\perp})$, $(\delta_i, \delta_i^{\perp})$ and h() pass through $(\epsilon_i, \epsilon_i^{\perp})$, $(\delta_i, \delta_i^{\perp})$

Once we confirm that the basic function h(x) has the desired properties, we fit this function with appropriate parameters to all regions. The final function is a smooth and continuous function, which passes through all target points $(\delta_{i_{SV}}, \delta_{i_{SV}}^{\perp})$, $(\delta_{i_{FV}}, \delta_{i_{FV}}^{\perp})$ as well as the two extreme boundary points as shown in Fig. 4.4.

In the training process, data samples with abnormal labels belonging to all patients in DS1 and the personalized normal cluster for one patient are used to determine the mapping function for the corresponding patient. In the predicting process, the feature vector of a new



Figure 4.3: Optimized piece-wise interpolated function p.

ECG sample in the hyper-spherical coordinate system is calculated and the mapping function is applied on its hyper-spherical coordinates to yield the transformed feature vector. After this step, we calculate the transformed data samples in the Cartesian coordinate system, which is further fed into the personalized classification stage defined by Eq. 2.8 to generate the corresponding type of *yellow alarm*.

4.6 Experimental Results

In this section, the performance of the proposed method is evaluated in terms of two aspects. We first analyze the classification performance of the system and then present the comparative results with respect to other representative ECG classifiers. Furthermore, the classification results are partitioned into two sets: *red alarms* generated by the *global classifier* and the final labels by combining the *yellow* and *red alarms*. In this way, the impact of *personalized classifier* on the final labels can be revealed. Finally, the prediction power



Figure 4.4: Optimized mapping function f.

of the proposed method in terms of providing precaution hints for upcoming *red alarms* is evaluated.

4.6.1 Classification Performance

The experimental results are assessed in terms of classification performance of 4 AAMI ECG classes using the test subset of MITBIH Arrhythmia DS2. Originally, DS2 contains 15357 samples after feature extraction. While training the *personalized classifier*, the normal samples within the first 5 minutes of the signal serve as initialization set for the personalized dynamic normal cluster. Therefore, we exclude the normal samples in the first 5 minutes of the signal from each record since they are already included in the training process. Consequently, the actual test set contains 12414 samples in total consisting of 10105 type-N, 1702 type-V, 508 type-S and 99 type-F samples.

To present the result, we select the weighted k-Nearest Neighbors method with k = 10 as our

		Ground Truth										
		Ν	V	S	F							
	Ν	9255(10076)	21(38)	72(90)	1(5)							
Result	V	657(22)	1678(1663)	8(2)	9(7)							
	S	71(6)	3(1)	417(416)	0(0)							
	F	122(1)	0(0)	11(0)	89(87)							

Table 4.1: Cumulated Confusion Matrix for All Records in DS2. The numbers are final label(primary label by global classifier).

choice of global classifier because it is one of the low-complexity methods with a relatively good classification accuracy [69]. The parameter α , used in Eq. 4.8 in the deviation detection module is set to 1 for test purpose.

Table 4.1 summarizes the cumulated confusion matrix for all records in the test set. In order to compare the result of global classifier (red alarms) and combined results (final labels including both red and yellow alarms), the sample numbers are presented in the following format: final label(primary label by global classifier). In order to measure the classification performance, we adopt three metrics: accuracy(Ac), sensitivity(Se), specificity(Sp), as proposed in [12,13,16]. All three metrics are calculated based on the true positive TP, false positive FP, false negative FN and true negative TN in a binary confusion matrix, where one class is the specific abnormality class and all other abnormality and normal classes combined into one class. Therefore all four metrics are calculated for each class by converting the 4x4 matrix to a 2x2 matrix.

While cumulative classification results are demonstrated in Table 4.1, the robustness of the proposed method should be evaluated based on the performance variation over 22 test records in DS2. Further, medians and IQRs (interquartile range) for each metric and each class are included in Table 4.2 to represent the robustness and average performance of proposed methods. The robustness of the system is assessed in terms of the variation between the performance of different records. The system is more robust if this variation is lower. In

etatietice	Ν		V		S			F				
Statistics	Ac	Se	Sp									
cumulated	92.4	91.59	95.93	94.38	98.59	93.71	98.67	82.09	99.38	98.85	89.9	98.92
median	94.45	92.21	95.42	96.17	99.55	95.71	99.38	80.65	99.84	99.11	90.91	99.11
IQR	6.33	10.08	11.91	5.17	1.64	8.62	1.76	19.35	0.61	1.58	23.33	1.49

Table 4.2: Classification Performance and Within-Set Variation of Proposed System

Table 4.3: V and S classification performance compared with five algorithms in literature using 11 common records in MITDB

Mathada	V			\mathbf{S}			
Methous	Ac	Se	Sp	Ac	Se	Sp	
Proposed	96.6	98.2	92.4	98.63	88.89	99.41	
Hu et al. [12]	94.8	78.9	96.8	N/A	N/A	N/A	
de Chazal <i>et al.</i> [9]	96.4	77.5	N/A	N/A	N/A	N/A	
Jiang and Kong [15]	98.8	78.9	96.8	97.5	74.9	98.8	
Ince $et al. [16]$	97.9	90.3	98.8	96.1	81.8	98.5	
Kiranyaz et al. [17]	98.9	95.9	99.4	96.4	68.8	99.5	

Table 4.2, we observe that among all abnormality classes, the proposed method demonstrates a stable performance on class V but a lower performance consistency for classes S and F.

As MITDB is widely used to verify ECG classifier performance, we compared the proposed system with five significant methods proposed in the literature. According to AAMI standards, the performance of ECG classification should be evaluated over the binary classifiers applied to *Ventricular* (V) versus *non-V* types and *Supraventricular* (S) versus *non-S* types. For methods proposed in the literature, the same evaluation metrics are commonly applied to records from MITDB. To standardize the metrics, we select 11 ECG records which are common among all 5 methods and compare the median of each classification metrics over these 11 records. The comparison results are presented in Table 4.3. Generally speaking, the proposed method shows a higher sensitivity for both types V and S. Especially for type S, the proposed method shows an advantage over all three metrics compared to the 5 reference methods.

4.6.2 Prediction Performance

As an important feature of the proposed method, yellow alarms triggered by the personalized classifier indicate higher probabilities of observing subsequent abnormalities of the same type. In order to verify this functionality, all beats following a yellow alarm of a specific type is investigated to asses the chance of upcoming red alarms of different types. This process is repeated for yellow alarms of all types. We only account for the first abnormality type which occurs after the yellow alarm. As we used confusion matrix to evaluate the classification accuracy, the performance of prediction can be summarized by a confusion matrix with the 3 abnormal types. The probabilities of observing a certain type of abnormal beat after a yellow alarm is calculated using the prediction confusion matrix and compared to the prior probability of observing the abnormality of the same type. This process is formulated in the following two equations:

$$P(\hat{y}_{k+i} = X_r | \hat{y}_k = X_y) = \frac{\# \text{ of } y_{k+i} = X \text{ after } \hat{y}_k = X_y}{\# \text{ of true alarms after } \hat{y}_k = X_y}$$

$$P(\hat{y}_{k+i} = X_r) = \frac{\# \text{ of true alarm of type } X (y_k = X)}{\# \text{ of all true alarms}}$$
(4.10)

The prediction power of each abnormality type is evaluated by comparing $P(\hat{y}_{k+i} = X_r | \hat{y}_k = X_y)$ and $P(\hat{y}_{k+i} = X_r)$. A shown in Table 4.4, the probability of observing a certain type of abnormalities after a *yellow alarm* is higher than its prior probability and this fact is consistent for all abnormality types. For example, without knowing the type of the preceding *yellow alarm*, the probability of observing a type V sample is 71.54%, while the probability of observing a type V sample after observing a *yellow alarm* of a type V is 77.45% (5.91% higher than the prior probability). The improvements are consistent among all three types

		#	of predicted		% of predicted		
		ground truth			ground truth		
		V	S	F	V	S	F
uellow	V	467	122	14	77.45	20.23	2.32
alarm	S	36	15	0	70.59	28.41	0
	F	40	60	5	38.10	57.14	4.76
tota	L	543	197	19	71.54	25.96	2.50

Table 4.4: predictive probability versus prior probability without windowing

Table 4.5: predictive probability versus prior probability within 10 beats' window

		#	# of predicted			% of predicted		
		ground truth			ground truth			
		V	S	F	V	S	F	
uellow	V	290	85	12	74.94	21.96	3.10	
alarm	S	22	13	0	62.86	37.14	0	
alaliii	F	29	37	6	40.28	51.39	8.33	
total		341	135	18	69.03	27.32	3.64	

of abnormalities but the system shows a stronger prediction power for type S.

In the above analysis we consider the first subsequent *red alarm* regardless of the time passes since the preceding *yellow alarm*. In order to study the impact of the timing window (the time between the *yellow alarm* and the subsequent *red alarm*), we also studied a window of 10 consecutive samples following a *yellow alarm*. Similarly, the prior and posterior probabilities are compared to evaluate the performance of the prediction capacity as shown in Table 4.5.

Compared with the result without windowing, the prediction performance within a 10-sample window shows that the proposed algorithm can better predict the occurrence of abnormalities if a certain timing window is used. Especially for type S, the probability of observing a sample of type S within 10 segments after a *yellow alarm* of type S is 27.32% (i.e. the posterior probability rises to 37.14%). With almost 10% increase, it is proven that the *yellow alarm* types are informative. The results show that same improvements are made within the 10-sample window as well. In general, the predicting performances are promising, indicating
the efficiency of *personalized classifier* and deviation analysis. We believe that this concept is worthy of further in-depth studies for different physiological signals.

4.7 Summary of Contributions

In this chapter, we proposed a novel deterministic spatial transformation. The purpose of this method is to reshape clustering geometry in the original feature space so that the clusters in the transformed feature space demonstrate symmetry and low within-cluster variance. This analytical method is more tractable than the method proposed in chapter 3. Further, it eliminates the need to seek for a proper kernel function or to heuristically optimize the coefficient of the selected function, which may require high computational complexity.

This method utilizes hyper-spherical coordinate system and the orthogonalization concept to determine the parameters of the basic functions used to implement the nonlinear mapping. We applied this method to the test dataset, MITDB DS2 and obtained promising results. Both the classification and prediction performances of the proposed method are analyzed in this chapter. The algorithm has been proven to be efficient in classifying samples and predicting upcoming abnormalities as well.

Chapter 5

Conclusions And Future Works

5.1 Conclusions

In this thesis, we considered an overlooked problem in the biomedical signal processing community, which is the lack of prediction power of automated biomedical signal processing methods in predicting severe health abnormalities ahead of time through processing mild abnormalities in the test signal. Several automated algorithms have been designed by the biomedical engineering community to process physiological signals to assist health providers in diagnosing different disorders and making better therapy plans. However, an important concept of predicting severe heart conditions before their occurrence by processing physiological signals is not well studied. In this work, we developed a novel methodology as a primary step towards developing predictive diagnosis tools. We applied the proposed methodology into ECG signals to assess the power of the system in predicting upcoming heart problems. In this regard, we proposed a patient-adaptive ECG classification framework. The system has a two-staged hierarchical structure including a *global classifier* and a *personalized classifier*. The *global classifier* is designed to filter out the samples with severe distortions represented by their abnormal waveforms by triggering red alarms. The samples classified as "normal" the by global classifier are delivered to the subsequent deviation detection module. In this stage, the *personalized dynamic normal cluster* is constructed and used to specify the normal range for each patient's ECG signal. By comparing a sample with its personalized normal range, we use two joint conditions to decide if the sample is in a fuzzy state between the normality and abnormality conditions. If the sample fails to meet both of these conditions, a *yellow alarm* is triggered to provide predictive information about upcoming abnormalities. The samples that show considerable deviations from their ground normal are passed to the *personalized classifier* to label it as one of the three abnormal types, whichever is more likely. On the other hand, the samples without considerable deviations are confirmed to be normal and are further utilized to update the *personalized dynamic normal cluster*.

In chapter 3, a *kernel*-based nonlinear transformation is proposed to manipulate the clustering topology in the original feature space. More specifically, a weighted combination of kernel functions are deployed in this method to implement a spatial transformation function. The desired topology is formulated in terms of two objective functions, so that the system is able to find the optimal coefficients of kernels by jointly optimizing these two functions. This non-convex multi-objective optimization is solved with a method based on MOPSO. In order to validate the improvement on spatial topology by introducing nonlinearities with kernels, we compared the Pareto front generated with the linear combination of the original features and the Pareto front produced in the transformed feature space with the utilized polynomial kernels. The results confirm that the kernel-based transformation allows more degree of freedom in optimizing the clustering topology according to the representative objective functions. Moreover, we applied this method to MITDB test dataset and obtained similar sensitivity and specificity results as proposed in the literature. More importantly, the predictive capability of *yellow alarms*, as a unique feature of the proposed method, is analyzed. The performance is quantified by comparing the prior and posterior probabilities of each abnormality class after observing a *yellow alarm* of the same type. The comparison results show that a promising improvement has been achieved by applying the nonlinear transformation.

While the method in chapter 3 demonstrates a capacity of predicting upcoming abnormalities of ECG signals, it remains challenging to interpret the mechanisms of the proposed system and thus hindering the generalization of the proposed predictive warning methodology to similar biomedical signal processing applications. Therefore, the main objective of chapter 4 is to develop a deterministic spatial transformation function, which is able to achieve the desired spatial topology with a more tractable analytical approach. Thus, we proposed a novel spatial transformation specifically designed to reshape the feature space according to angles between cluster centroids using the customized inverse of *logit* functions. In this method, the between-cluster cosine distances are optimized through orthogonalization of cluster centroids using spherical coordinates. Meanwhile, the within-cluster variance is reduced by a piecewise mapping function consisting of the previously designed basis functions. The basic function proposed in chapter 4 has the property of saturating at the boundaries, which is similar to the inverse of logit function but is yet more flexible. An advantage of deploying such basic functions is that the clustering geometry is preserved after spatial transformation. We implement this novel transformation in the patient-adaptive classification framework. The performance of this system is evaluated with classification and prediction results on the test dataset. The classification results show that by triggering *yellow alarms* through this method, the *specificity* as well as the performance consistency of abnormal types is improved compared with the method proposed in chapter 3 as we can observe higher *specificities* and lower IQRs. The improvement on type V (ventricular) is most notable, as the median of accuracy is improved from 86.11% to 96.17% and the IQR is lowered 22.37%. Comparing with the methods in the literature, the proposed method in chapter 4 is remarkable as well. Especially for the type S (*supraventricular*), the proposed system performs better in identifying class S than all 5 similar benchmark methods reported in the literature.

We also studied the impact of the time lag between a *yellow alarm* and the subsequent real abnormality in chapter 4. The results show that most of the real abnormalities occur within 10 samples after a *yellow alarm*. Overall, the system has been proven to be efficient both in classification and prediction aspects in this work. In short, this study suggests that predictive modeling of physiological signals can be used as alarming hints for upcoming health conditions, which can have a wide range of applications in developing wearable biosensors, automated diagnosis tools to assist patents and physicians in predicting health problems, This methodology have a great potential to impact the emerging research fields of smart health and smart cities.

5.2 Future Works

In this research, we focused on improving two main drawbacks of automated ECG analysis in the literature, namely, the failure in capturing the inter-patient variability and the incapability of early detection and prediction. We proposed two methods for improving predictive capability. While the results show the promising performance of the designed system, further investigations can help to improve and generalize the proposed system to other types of biomedical signals. The following tasks can be considered as some future directions enabled by this research:

• investigate other kernel functions to improve the transformation for spatial topology optimization;

- seek deterministic solutions and develop more systematic performance analysis for the objective functions used to assess the *separability* and *symmetry* of clustering geometry in transformed feature space;
- assess the performance of the proposed spatial transformation on other biomedical signals with similar properties, such as EEG, EMG, and EOG;
- improve the deterministic mapping function by including the size of clusters in the mapping function;
- integrate the measurements from other wearable body sensors (such as accelerometer, temperature etc.) in the monitoring system to capture the impact of environmental condition on temporal variation of signals;
- design a mechanism to correct the false *red alarms* triggered by *global classifier* and further improve the classification performance.

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